Radiative heating of circumstellar disks

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1. Motivation

Heating by stellar irradiation plays a crucial role in the energy balance of circumstellar disks. We developed a model to parametrize radiative heating of an optically thick disk in the hydrodynamics code Fosite (Illenseer and Duschl, 2009). To be able to simulate long term disk evolution, we keep our model as computationally efficient as possible. This forces us to make some simplifying assumptions. That is why we test our model by comparing snapshots of Fosite simulations to solutions obtained with the the Monte-Carlo 3D continuum radiative transfer code MC3D (Wolf et al., 1999, 2003).

2. Radiative heating and cooling

Radiative heating

Following Chiang and Goldreich (1997) we assume that radiative heating by the central star takes place in a thin superheated surface layer of a flared accretion disk. Its calculation is based on the work of Kenyon and Hartmann (1987):

$$Q_{\text{irr}} = \epsilon\alpha\sigma_{\text{B}}T_*^4\left(\frac{R_*}{r}\right)^2$$

T_* : Effective temperature of the star
R_* : Midplane temperature
σ_{B} : Stefan-Boltzmann constant
α : Angle between the disk surface and the stellar radiation
ϵ : Scattering efficiency

To calculate the angle α we need the height over the midplane where the optical depth of the disk material becomes unity for radiation coming from the central star (see Sections 4.,5.).

Radiative cooling

We compute the radiative energy loss of the disk by treating its surface as a block body:

$$Q_{\text{cool}} = 2\sigma_{\text{B}}T_{\text{eff}}^4 = 2\sigma_{\text{B}}\tau_{\text{eff}}T^4$$

T_{eff} : Effective temperature of the disk surface
T : Midplane temperature
τ_{eff} : Effective optical depth (Günther et al., 2004)

3. Hydrodynamics

We describe the disk evolution using the vertically integrated equations of compressible, viscous gas dynamics. Those are the continuity, the Navier-Stokes and the energy equation:

$$\partial_t\Sigma + \nabla\cdot(\Sigma\mathbf{v}) = 0$$

$$\partial_t(\Sigma\mathbf{v}) + \nabla\cdot(\Sigma\mathbf{v}\otimes\mathbf{v} + \Pi\mathbb{1}) = \nabla\cdot\mathbb{T} - \Sigma\nabla\Phi$$

$$\partial_t E + \nabla\cdot((E+\Pi)\,\mathbf{v}) = \nabla\cdot(\mathbf{v}\cdot\mathbb{T}) - \Sigma\nabla\Phi\cdot\mathbf{v} + Q_{\text{rad}}.$$

Σ : Surface density
$\mathbb{T}$: Viscous stress tensor
Φ : Potential of the central star
Q_{rad} : Radiative heating and cooling
$\mathbf{v}$: Velocity
Π : Pressure
E : Total energy

To close this system of coupled differential equations we assume that the disk material can be treated as an ideal gas.

4. Model assumptions

- Geometrically thin, non self-gravitating disk
- Dust dominates absorption
- Dust to gas ratio $1:100$
- No spatial dependency of dust absorption
- Wavelength dependency of the dust properties is accounted for by using the wavelength defined by the maximum of the product of the stellar SED and the dust absorption efficiency
- Vertical density structure does not deviate significantly from a vertically isothermal structure (see Section 6.)

5. Height calculation and flaring angle

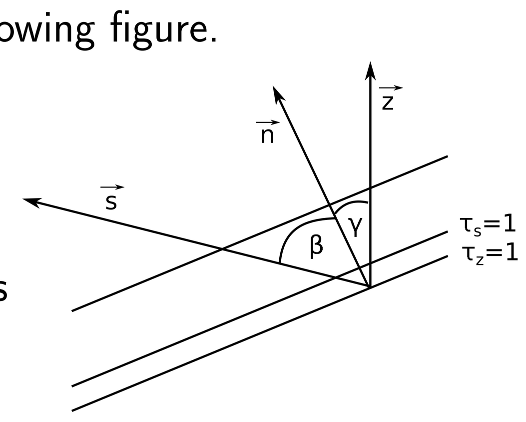
Using the model assumptions we can calculate the height H where the disk becomes optically thick for stellar radiation:

$$H = \sqrt{2}\,h\,\text{erf}^{-1}\left(1 - \frac{\cos\beta}{\cos\gamma}\frac{2}{\Sigma\kappa}\right) \qquad \kappa = \frac{3Q_{\text{ext}}\sigma_{\text{geo}}}{4\rho_{\text{dust}}\pi a_{\text{eff}}^3}.$$

h : Pressure scale height
Q_{ext} : Extinction efficiency
ρ_{dust} : Dust density
Σ: Surface density
σ_{geo}: Scattering cross section
a_{eff}: Effective dust grain radius

The angles are defined in the following figure.

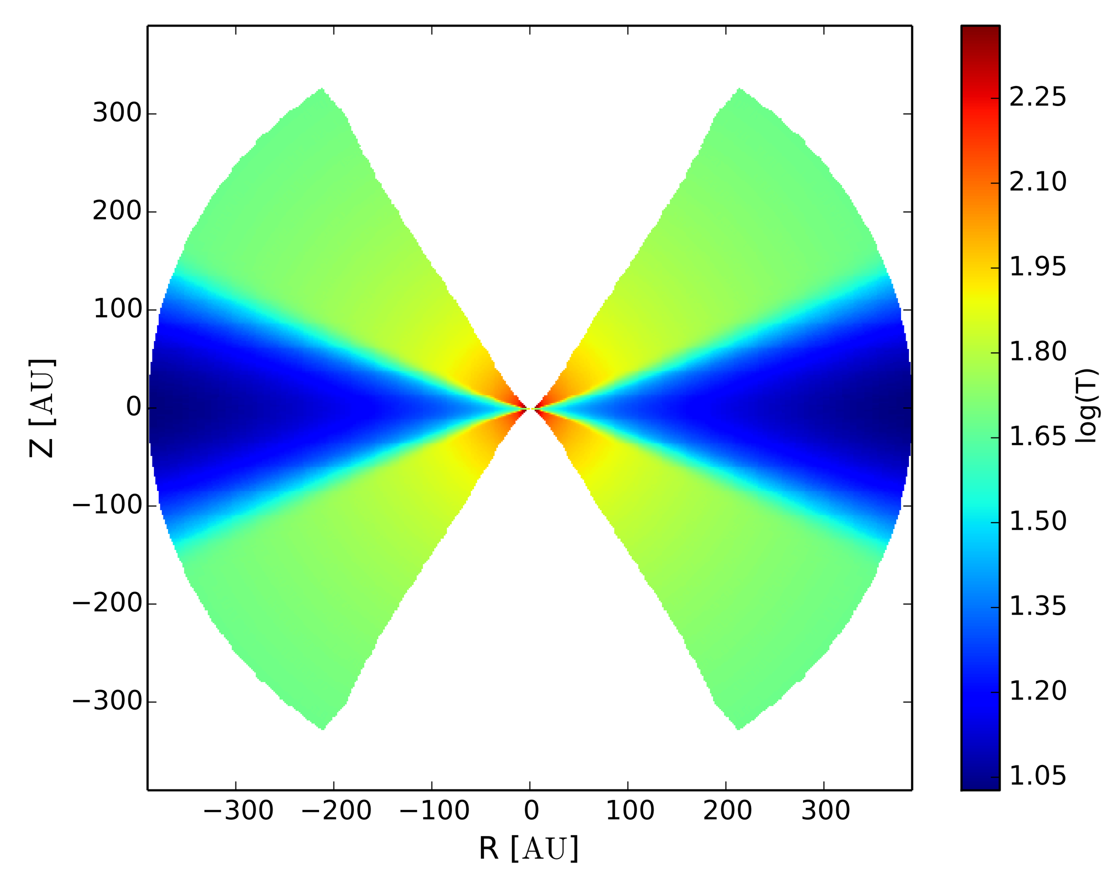
$\vec{s}$: Direction to central star
$\vec{n}$: Surface normal
$\vec{z}$: Vertical direction
τ_z, τ_s : Surface of constant optical depth for photons traveling in negative $\vec{z},\vec{s}$ direction



Knowing the height of the disk we can calculate the angle α from Section 2. Self-shadowing is taken into account by setting $\alpha = 0$ for back faces of elevated structures.

6. Vertical structure

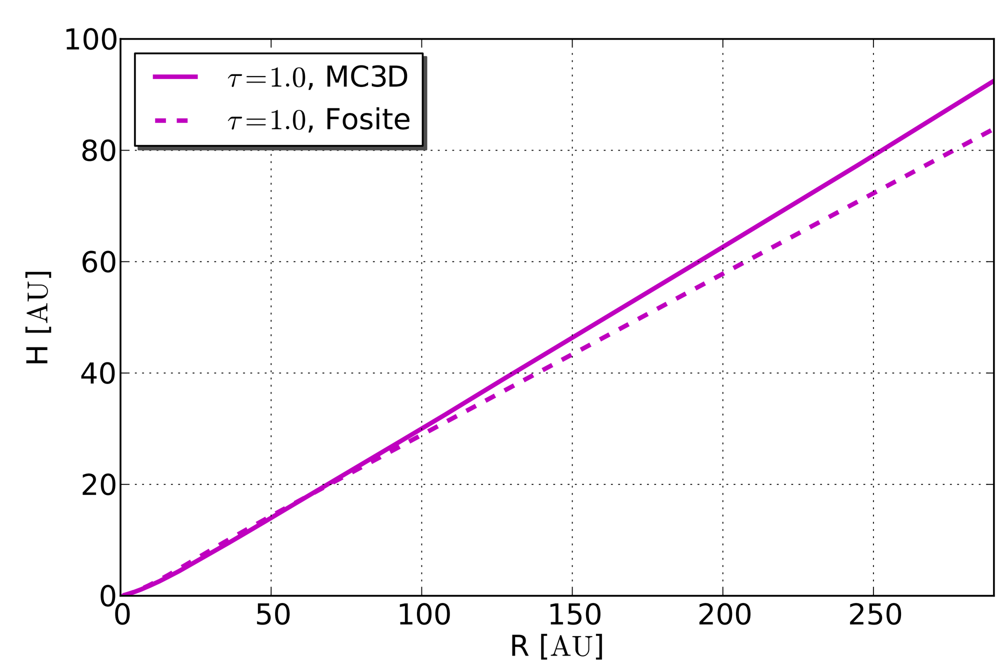
We test the assumption of an isothermal vertical structure using 3D temperature maps of our disk models obtained with MC3D.



Vertical cut through the 3D disk temperature structure

The cold, nearly isothermal part corresponds to the regions of high density. As those regions define the midplane temperature which is the only relevant temperature for 2D disk hydrodynamics, we can use the vertically isothermal disk structure.

7. Disk height



Height corresponding to an optical depth $\tau = 1$

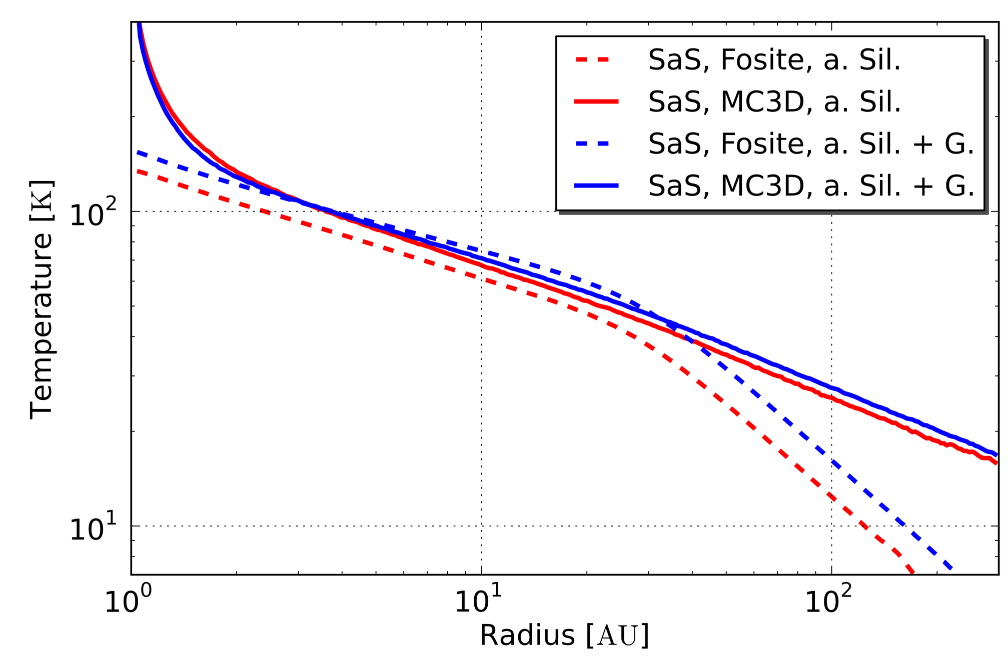
Calculating the correct disk height is crucial for radiative heating. That is why we compared the solutions obtained by Fosite to those computed with MC3D. Fosite results are shown in dashed and MC3D results in solid lines. Fosite results are in very good agreement with the MC3D values.

8. Results

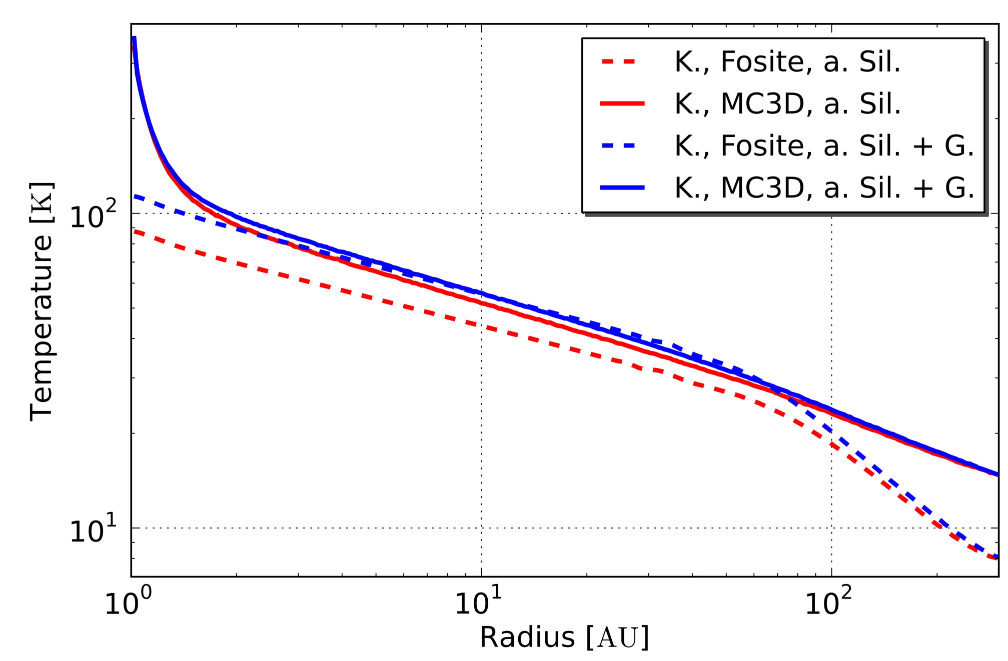
To test our heating model we compared midplane temperatures calculated by Fosite and MC3D for different initial density distributions, dust compositions and disk masses of $0.01M_\odot - 0.001M_\odot$.

Density distributions are based on the work of:
SaS: Shakura et al. (1973), parameters: Gräfe et al. (2013)
K: Kalnajs (1972)

Chosen dust compositions:
a.Sil.: Astronomical silicate (Weingartner et al., 2001)
a.Sil.+G.: $62.5\,\%$ astronomical silicate $+\ 37.5\,\%$ graphite



MC3D and Fosite midplane temperatures for Shakura Sunyaev disks for both dust compositions



MC3D and Fosite midplane temperatures for Kalnajs disks for both dust compositions

9. Conclusions

- Computationally efficient approximation for radiative heating
- Sensitive to the chosen chemical composition, grain size, density and extinction efficiency of the dust
- Temperature structure in the dense, central disk regions is in very good agreement with 3D radiative transfer simulations
- Deviations from the correct temperature for small radii are caused by different boundary conditions and the fact that Fosite uses only one wavelength for heating
- Deviations for low density regions at large radii can most likely be explained by the fact that Fosite ignores radial photon transport

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