CONSTANT ELECTRIC CURRENT AND THE DISTRIBUTION OF SURFACE CHARGES[1]

Hermann Härtel

Guest scientist at

Institute for Theoretical Physics and Astrophysics

University Kiel

ABSTRACT

Surface charges are present, whenever a wire carries an electric current and are necessary for a causal explanation of how an electric current actually works. It could proof to be helpful for students understanding, if the existence and necessity of surface charges would be introduce into the physics curriculum not only when teaching electrostatics but also when treating electric circuits.

DEVELOPMENT OF SURFACE CHARGE

In the following diagram, a battery is used as a device to separate charges. This separation is performed by a chemical process resulting in a state of equilibrium with a certain number of electrons on the surface of the negative metallic outlet and the same number of electrons missing on the surface of the other side.

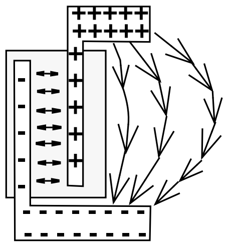

Fig. 1: Charged Battery

A dynamic equilibrium is reached when the attracting Coulomb forces between these separated charges are equal to the separating chemical forces.

If two conductors and a resistor are connected to the battery, an electric current starts to flow. There are also surface charges spreading out all over the circuit within very short time. The way in which these surface charges are distributed is represented in the next picture (in a rather crude and exaggerated form).

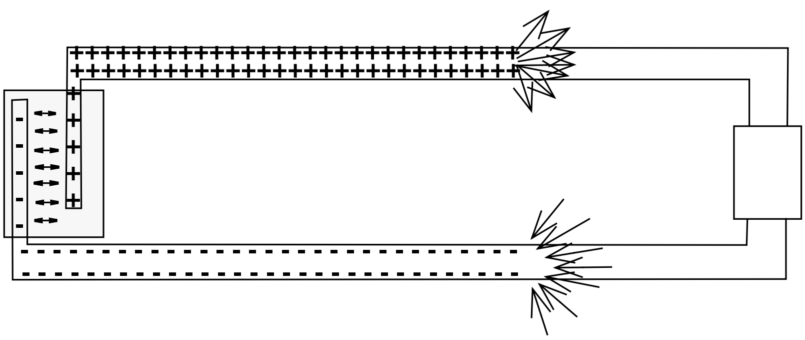

Fig. 2: Development of Surface Charges - Intermediate State

1. This is a revised part of a full paper, published under the title "A Qualitative Approach to Electricity" at: Institute for Research on Learning, Report No. IRL87-000; 1987

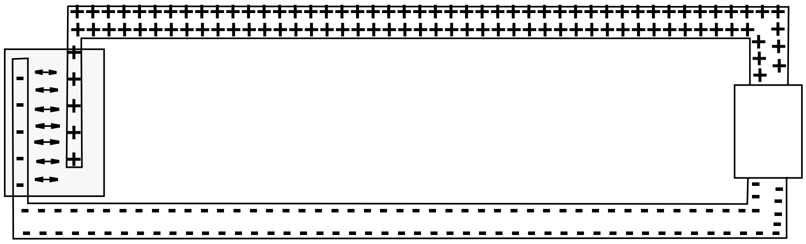

Fig. 3: Development of Surface Charges - Final State

The existence of these surface charges and their interaction through space can be demonstrated with the following experiment.

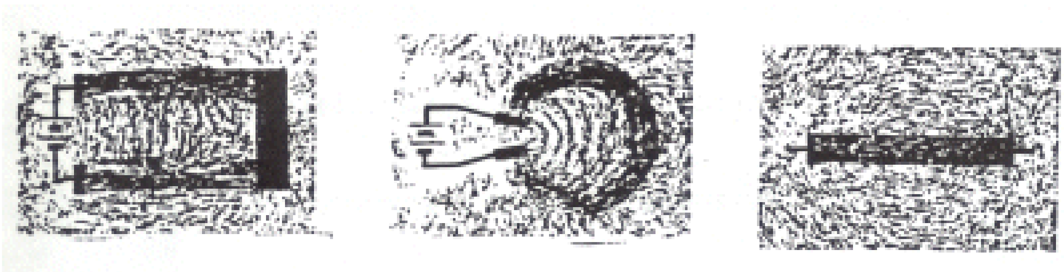

Fig. 4: Electric Field Lines Around Conductors

For this demonstration, the circuits are made out of conductive ink and placed on glass plates (Jefimenko,1963). The change in space around the circuit due to the surface charges is demonstrated with the aid of grass seeds strewn upon the glass plate. Grass seeds orient themselves in the neighbourhood of charges because they act like electric dipoles (analog to magnetic compass needles).

GRADIENT IN SURFACE CHARGES

In order for the students to understand the importance of these surface charges in respect to current flow and voltage, the distribution of these charges has to be studied in detail. The following points are of interest:

- order of magnitude of these surface charges
- distribution along a linear conducting wire
- influence of symmetry and distance

Order of Magnitude - If a current of 1 A is flowing through a wire, there are about 10^{19} electrons passing through each cross section within one second. Because of the enormous strength of the Coulomb interaction and the very high mobility of electrons in metals, it takes only a few electrons at the surface of the wire to push 10^{19} electrons around in a circle and to overcome the resistance of a metallic wire.

Distribution of Electrons on the Surface - For reasons of symmetry, it can be concluded that the interaction of equally distributed electrons on the surface of a round wire with electrons inside the wire cancels to zero. For every small area A containing a certain number of electrons, there exists another area A' at the opposite side with the same number of electrons which will balance the interaction with some electron at point P.

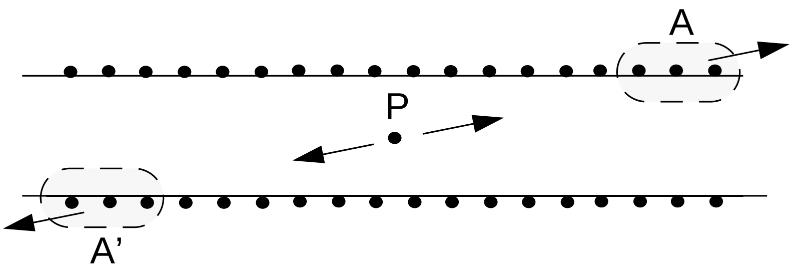

Fig. 5: Influence of a Constant Surface Charge (for Points P on the Axis)

This symmetry argument is exactly true only for points P on the centre axis of the wire. For other points off axis, P', the symmetry argument only holds for forces parallel to the axis of the wire.

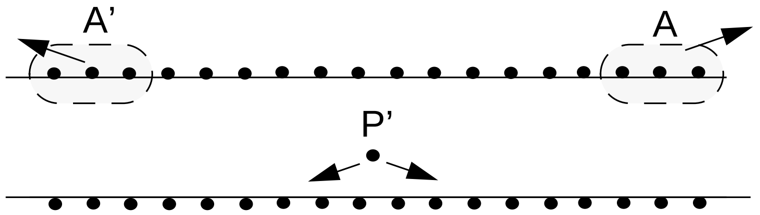

Fig. 6: Influence of a Constant Surface Charge (for Points P' off Axis)

The forces perpendicular to the axis, however, have to be zero because otherwise the electrons would move to the surface and redistribute until there is no longer a force towards the surface.

To break this equilibrium between equally distributed surface charges and to produce a driving force for the electrons within the wire, it is necessary to change this symmetric distribution. On one side, there have to be more electrons than on the other side, so that the two interactions of opposite areas A and A' do not balance.

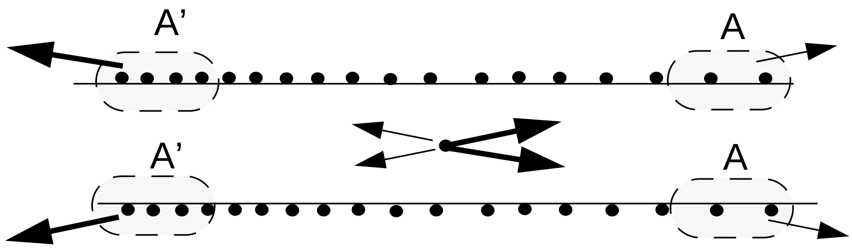

Fig. 7: Gradient in Surface Charge Distribution

*Influence of Symmetry and Distance -*The electric interaction between charges is dependent inversely on the distance between them. A ring of surface charges should, therefore, have the strongest interaction with the free electrons within the same cross-section. Because of the circular symmetry of the wire, however, the interaction of the surface charges cancel each other, and there is no net force on electrons situated at the same cross-section.

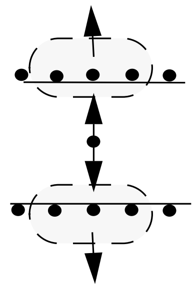

Fig. 8: Influence of Opposite Surface Charges

When moving away from this cross-section the components in the direction of the wire do not cancel any more; and the resulting forces increase. They reach a maximum at a distance twice to five times larger then the diameter of the wire. At larger distances, the interaction vanishes rapidly because the distance increases.

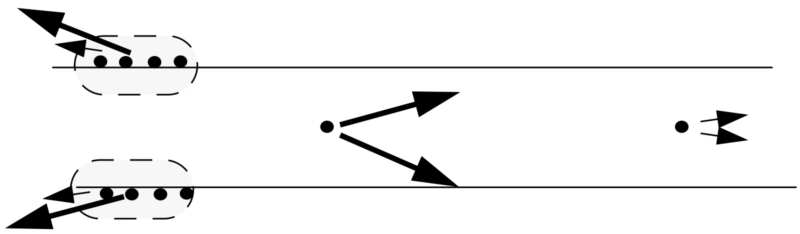

Fig. 9: Influence of Surface Charges at Different Distances

It is, therefore, sufficient to study the influence of a particular surface charge at distances comparable to the diameter of a wire.
For a long and thin wire, it can be shown by calculation that a uniform decrease of surface charges in the direction of the axis (a surface charge with a constant gradient in X-direction) results in a uniform and constant force for electrons in the direction parallel to the axis of the wire. (Walz 1984)
If the wire is curved or if other wires with surface charges are close together, the gradient is no longer constant and the circular symmetry of the charge distribution around the wire may be broken.

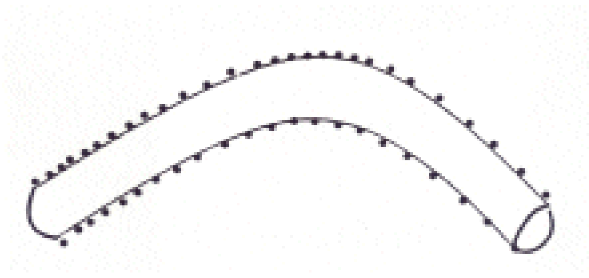

Fig. 10: Non-linear Distribution of Surface Charges

There have to be more electrons at the outer part of the curve than at the inner side to change the direction of the moving electrons inside the wire.
Surface charges are, therefore, only a qualitative indication for voltage. They cannot serve for a quantitative definition since they can only be determined for rather simple arrangements. The general and quantitative definition of potential difference or voltage is therefore taken as "energy/charge".

CONDUCTION MECHANISM AND RESISTORS

The mobility of electrons in metals like copper is many orders of magnitude larger than in a resistor. Within the copper wire, the electrons experience a very small force; and it is possible to think of a mechanism of conduction where the single electrons are accelerated under the influence of the interaction with the surface charges and collide with some atoms within the lattice. The number of collisions per second is high (about 10^{14}/s), and the mean free path between two collisions is about 10 diameters of a copper atom. In this theory about the mechanism of conduction in metals, the electrons can, therefore, be regarded as independent particles which are accelerated for a short moment and than stopped by collision. In this approximation, the interaction between the free electrons is neglected. In equilibrium, when the distribution of the surface charges is stable and the electric current is constant, this approach is correct because the interaction and the enforced increase of the velocity of the electrons is so very small. Therefore, the density of the electrons is practically not changed, and the demand for neutrality is always fulfilled.

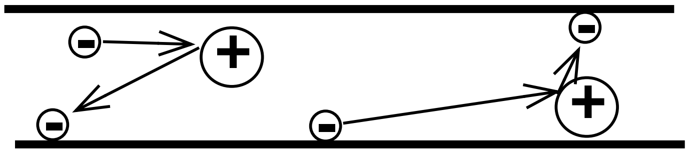

Fig. 11: Conduction Mechanism in Metal

The function of a resistor is, in general, to hinder the flow of charge carriers and to exchange energy. There are different ways to accomplish this function. One possibility is to reduce the diameter of the conducting wire without changing the material.

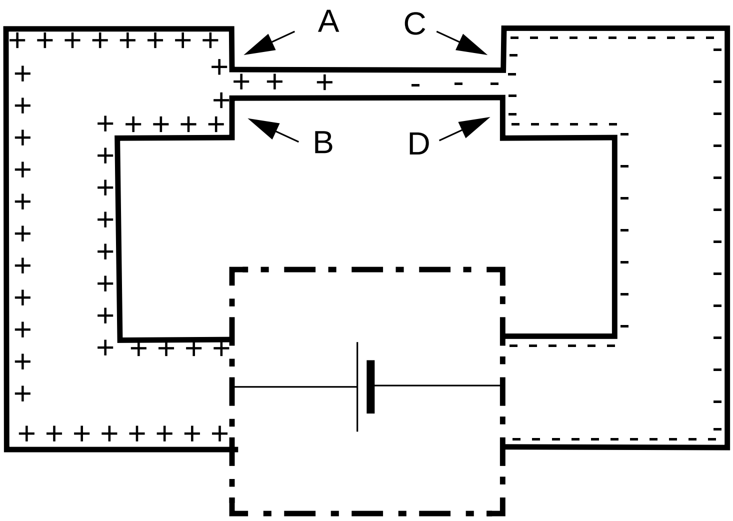

Fig. 12: Resistance Through Reduction of Cross-section

The effect of resistance in respect to electron flow can easily be explained through a geometric arrangement. The surface charges on the areas A, B, C, D which always exist in the presence of a changed cross-section, will oppose the effect of the original charges on the battery terminals and will, thus, reduce the intensity of the current flow in the conductor.

The gradient of the surface charge distribution is larger along the thin wire than along the thick ones. Therefore, a stronger force is exerted on the free electrons

within the resistor leading to a higher drifting speed. The longer the thin wire, the smaller this gradient will be and this will lead to a smaller current.
When equilibrium is reached, the same number of electrons will pass through any cross-section of the system in any given line. The drifting speed, however, will be different at different cross-sections.
In most practical cases, a resistor is introduced by choosing a different material. Such a material possesses a higher density of obstacles to electron flow, and/or it offers a smaller concentration of free electrons. Both effects normally appear together within a resistor and can cause a dramatic change in mobility for the electrons.
If a uniform conductor and a uniform resistor are soldered together, there exists two thin areas or better volumes within which the change of conductivity occurs. These areas are represented in the next picture on an exaggerated scale.

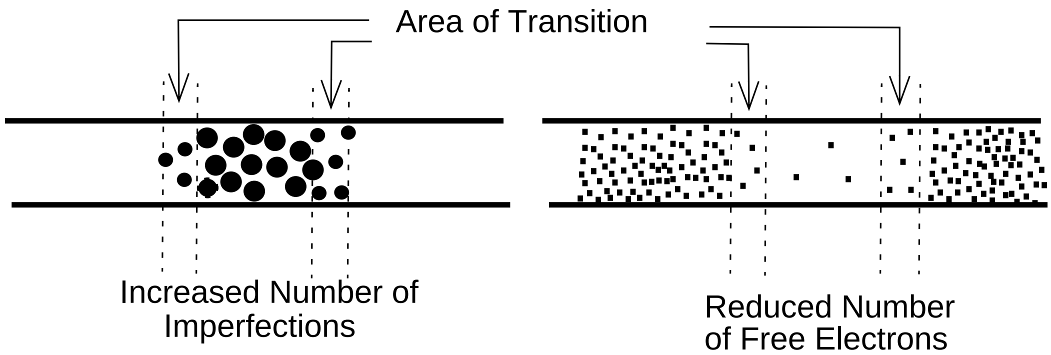

Fig. 13: Different Mechanisms for Electric Resistance

In the left picture, the filling represents the density of obstacles for the moving electrons. In the right picture, the density of the free electrons is represented. In both cases, the function will be the same. Within the area of transition from high to low values for the mobility, there will be on one side a slightly higher concentration of electrons than neutral and a slightly lower one on the other side. These extra charges, which are spread out within two thin volumes, will have two effects: On one hand, they oppose the flow in the conductors and on the other, they increase the force on the free electrons within the resistor. In dynamic equilibrium the same number of electrons will flow through any cross-section in any given time. Due to the extra charges and the different gradient of the surface charge distribution, the driving force within the conductor and the resistor are, however, quite different.
To treat the effect resulting from these transition areas seems to be important for two reasons. First, the principle that nature changes continuously in space and time is basic for qualitative reasoning. Therefore, the dramatic change in mobility should not be treated as a step function with no further consequences. Second, this area of transition plays an important role when the conduction mechanism in semiconductors and the function of diodes and transistors have to be explained.
A last possibility to resist an electron flow is through magnetic interaction from the outside. In this case, it can be assumed that an additional force from outside is holding back parts of the electron distribution within the circuit. This leads to a redistribution of the surface charges within the other parts just as if an ordinary resistor would have been introduced.

For any of these different resistors, a similar surface charge distribution will occur which is represented in the following picture.

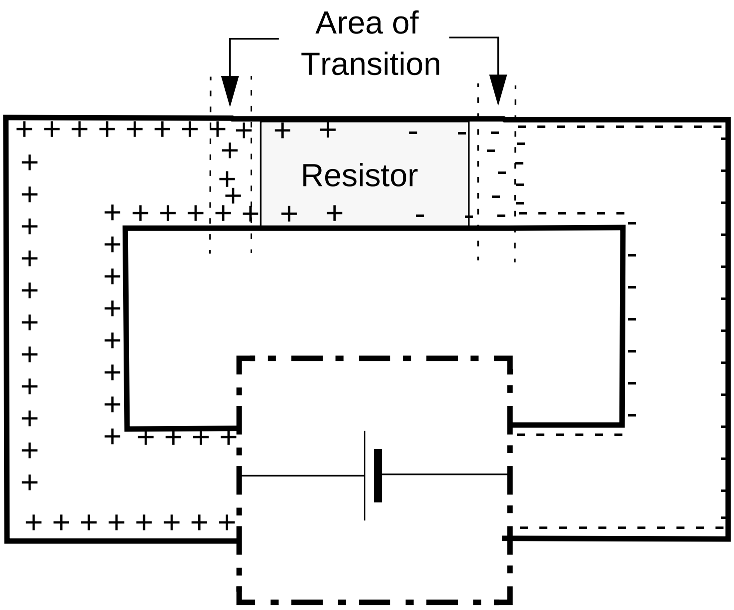

Fig. 14: Surface Charge Distribution (Circuit with one Resistor)

This picture indicates the principal ideas, it does not represent reality in all respects. The scale of the different parts is chosen to optimize visibility, not to represent reality on a correct scale. Moreover, the gradient in the surface charge distribution on the copper wires is not shown because it is too small compared with the gradient along the resistor.

A representation of the surface charge distribution in series and parallel circuits is shown in the following picture.

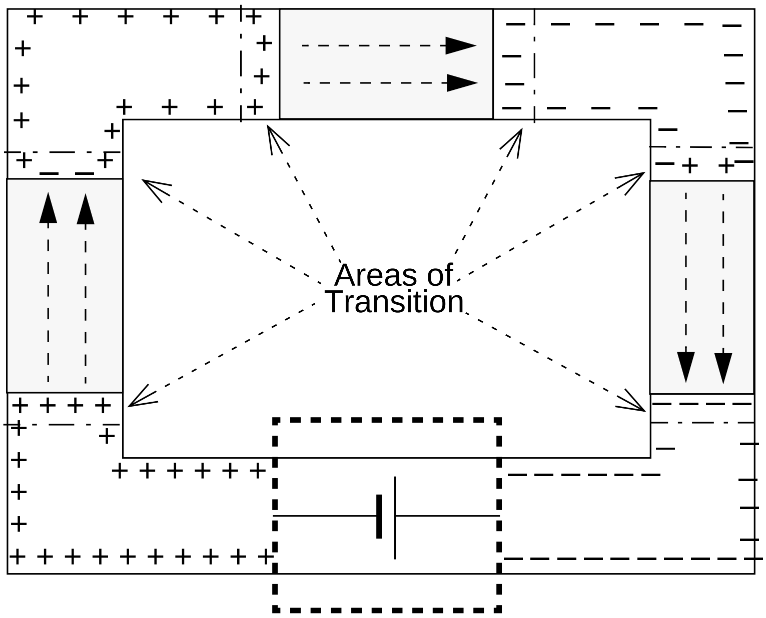

Fig. 15: Surface Charge Distribution in a Series Circuit

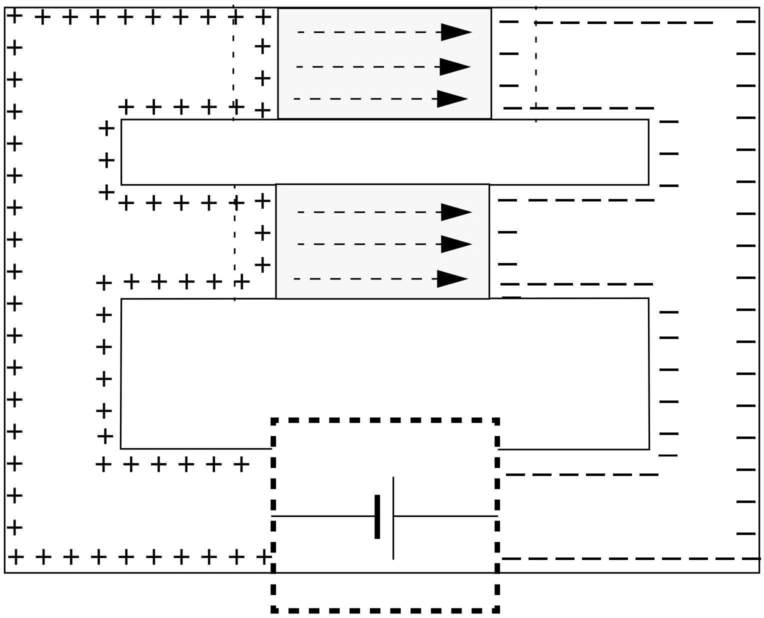

Fig. 16: Surface Charge Distribution in a Parallel Circuit

REPRESENTATION OF VOLTAGE

A complete circuit with an indication of surface charges could be represented like in figure 17 on the left.

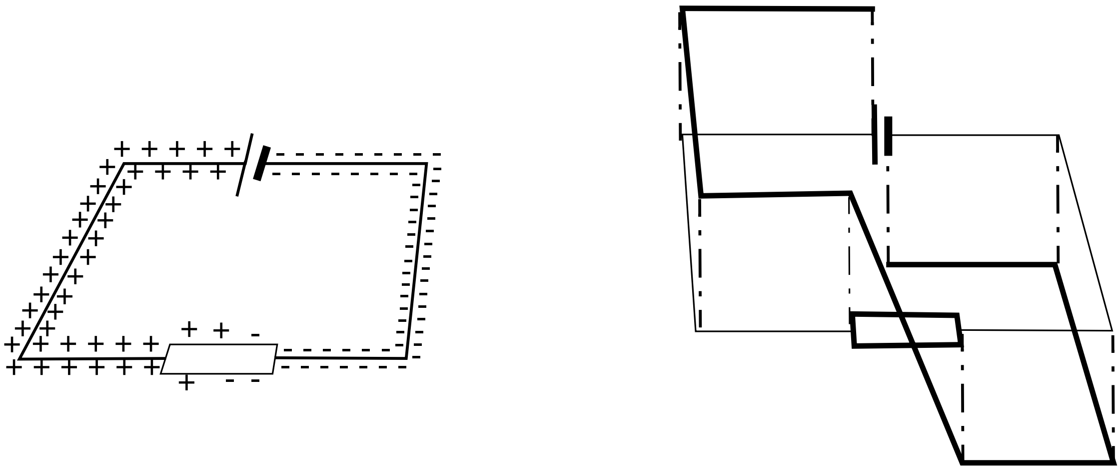

Fig. 17: Different possible Representations of Voltage

Once the principle of the distribution of the surface charges is understood, a different representation could be chosen by representing different values of surface charges - or better: different values for potential differences - by vertical lines of different length. In fig. 17 on the right, positive surface charges are indicated as lines in the upper direction and negative charges as lines downwards. This representation is more abstract but better to read, and the gradient of the surface charge distribution (potential difference) is immediately visible. The same information can be presented by choosing one side as zero (same height as the circuit).

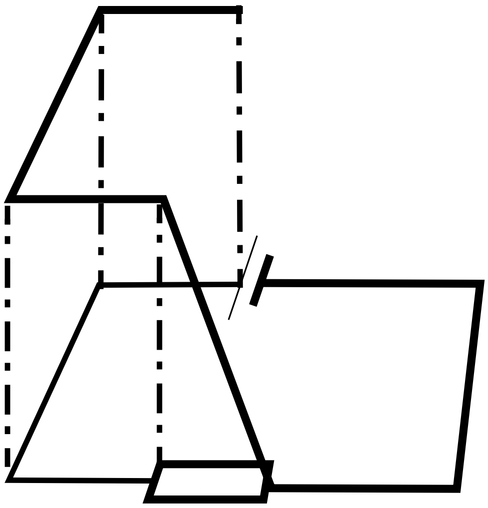

Fig. 18: Representation of Voltage (Change of Reference Point)

It is not the absolute value of the surface charges (potential) but only the difference in density between two points (potential difference) that causes a net driving force on the mobile electrons within the conductor and the resistor.