THE ROLE OF MATHEMATICS FOR PHYSICS TEACHING IN THE AGE OF COMPUTERS

Hermann Haertel
IPN - Institute for Science Education, D-24098 Kiel, Germany

The basic laws of Nature can be represented in mathematical form. This fact makes physics a prototype of exact science and is primarily responsible for its predictive power. However, a difficulty faces newcomers learning physics. Mastering mathematical formalism is often a prerequisite for understanding, and for many students this formalism acts as a barrier, too high to be overcome. For a range of different topics, such as free fall, rotation of rigid bodies, forced oscillation and wave transmission, it will be shown how this barrier can be avoided by using the results of numerical algorithms in the form of dynamic and interactive visualizations. Such visualizations are produced using advanced simulation programs. Examples will demonstrate how this approach allows the building of a conceptual understanding before mathematical formalism is introduced. A fundamental question for discussion remains: what kind of formalism and closed form solutions are essential, in view of our ability to readily generate numerical solutions?

1. Traditional approaches

Gravity and Inertia

On the base of Newtons law F=m*a and the law of gravitation the equation for the free fall close to the surface of the earth is derived as:

$F= m \gamma M / r^2 = mg = m a.$

To cancel m on both sides some words have to be lost about the fact that inert mass and gravitational mass are always proportional to each other. Since both are measured in kg, the term m can be cancelled.

In most class rooms the experiment is shown, where the feather and the piece of lead are dropping down within a tube with and without air.

Nevertheless, teachers experience again and again that most students don’t change their pre-concepts about dropping bodies. Heavy object drop faster than light ones and physics teaching seems to have little impact.

Rotating stick

If a stick in an inclined position is dropping down, the stick is rotating around its fixed end point P.

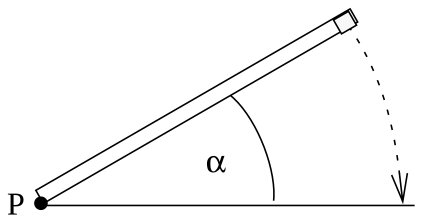


Figure 1 Stick, dropping down while rotating around P

The acceleration of the upper part, moving on a circular path, is larger than g during the final period of the fall. The mathematical proof is as follows:

The equation to be applied is:

$M = \Theta\, d^2\alpha/dt^2$

Inserting the moment of inertia Θ for a uniform stick and the applied torque for the centre of mass we get

$mg\, L/2 \cos \alpha = mL^2/3\, d^2\alpha/dt^2$

$d^2\alpha/dt^2 = 3g/2L \cos \alpha.$

The tangential acceleration a_{tang} of the upper end point of the stick is:

$a_{tang}=L\, d^2\alpha/dt^2 = 3/2\, g \cos \alpha$

Its vertical acceleration is

$a_{vert} = a_{tang} \cos \alpha = 3/2\, g (\cos \alpha)^2$

The special case “$a_{vert} = g$” leads to

$(\cos \alpha)^2 = 2/3$ and $\alpha = 35^o$.

It can be concluded that for α between 0 and 35^o the vertical acceleration of the end point is larger than g. As a student I can be impressed by the mathematical power of this approach. However, I do not get an answer to the question about the causality behind this process. There must be an extra force besides gravity. Where does the extra force come from? The mathematics above does not reveal an answer.

Forced oscillation

In mechanics and even more in electricity the oscillating system driven by an alternating external force is a classical topic, and an excellent example to demonstrate the usefulness and power of differential equations and their closed form solutions.

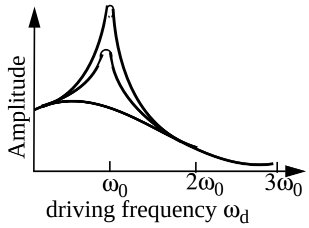
The starting equation is:

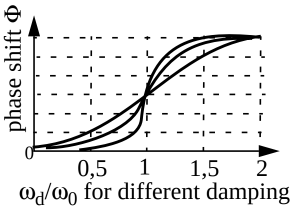
$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \cos\omega_d t$$

The solution is:

$$x_0 = \frac{F_0}{m \cdot \sqrt{\left(\omega_d^2 - \omega_0^2\right)^2 + b^2 \cdot \omega_d^2 / m^2}}$$

$$\tan\varphi = \frac{\omega_d b}{m\left(\omega_0^2 - \omega_d^2\right)}$$





There is no doubt that this is a wonderful example for the power of applied mathematics. However, the solutions are presented in frequency domain with the phase shift as the second important factor. This is one level above time domain, where the interaction actually happens and cause and effect can be analysed. The question remains, how much inside a student will acquire, even if a series of experiments has been carried out.

Wave transmission in one dimension

There are different approaches to arrive at the basic equations for the description of wave transmission in linear system.

A common approach starts with a double line made up of lumped elements, where Kirchhoff's laws are applied to each knot and each mash.

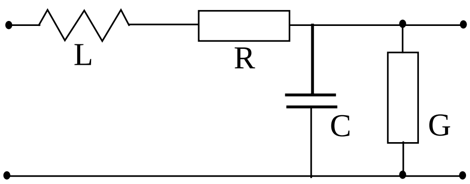


Figure 2 Basic element of a double line

$$\frac{\partial}{\partial x} i(x,t) = -C \cdot \frac{\partial}{\partial x} v(x,t) - v(x,t) \cdot G$$

$$\frac{\partial}{\partial x} v(x,t) = -L \cdot \frac{\partial}{\partial x} i(x,t) - i(x,t) \cdot R$$

Telegraph equations

The solution of this system of partial differential equations of first order (the telegraph equations) is given as combination of sin and cos expressions. With given initial conditions the transmission of waves in one dimension can be described.

Again it can be asked, how much inside a normal student will acquire from this approach? How much conceptual understanding about reflection and absorption for matched and unmatched lines will be found?

2. The alternative: use of computer simulations

Gravity and inertia

At IPN a simulation program was developed with a broad spectrum of possibilities to simulate interacting particles [1]. This program allows to replace gravitational mass with charge and Coulomb forces can be applied instead of gravity. In such a "world", objects of different inert mass experience a different acceleration, if the ratio of charge and mass is not the same (Figure 3).

The same idea can be realised with orbiting satellites (Figure 4) or with swinging pendula (Figure 5).

Students can be guided through these different simulations, where charge and inert mass can be varied independently.

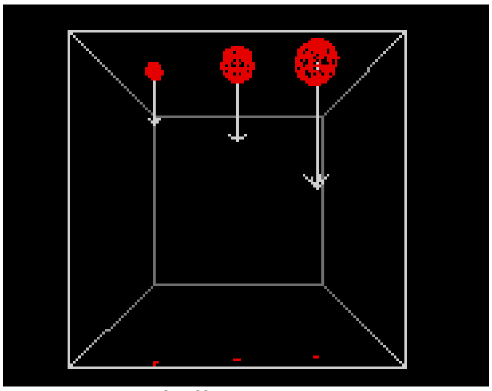


Figure 3 Objects of different mass and charge in a vertically oriented electric field.

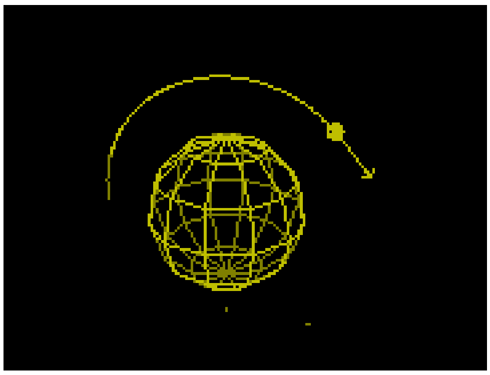


Figure 4 Satellite orbiting a central object. Gravity is replaced by Coulomb forces

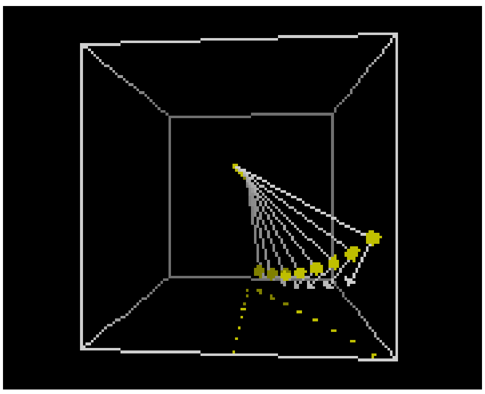


Figure 5 Within an electric field, charged pendula swing with the same frequency only, if the ratio of charge and inert mass is the same.

By varying these parameters the students can detect that the ratio of charge and inert mass is important to predict, if the mass will have an influence or not. This result is then transferred to the real world, where the ratio of inert mass and gravitational mass is always constant.

First tryouts with a small group of students have shown quite promising results. A study with a larger population is planned.

Rotating stick

Within xyZET a simulation can be constructed, where a comparison between a free falling body and a dropping (rotating) stick is shown. The stick is modelled as an array of mass points connected by springs. For starting angles less than about 48^0, the stick touches the floor first [2]. By varying the strength of the connections between the mass points, it can be shown, that bending of the stick and the additional elastic forces are the reason for this effect, even though no bending may be any more visible. An interesting discussion may follow about the question, why a theory, based on the model of the rigid body excludes elasticity, which is the main cause and nevertheless agrees perfectly with the experiment.

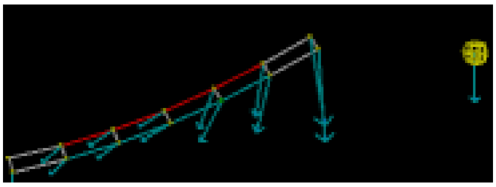


Figure 6 An elastic stick bends when falling down. Due to the additional elastic forces, the upper end is accelerated faster than a free falling object.

The forced oscillation

Experiments in the field of forced oscillations are limited by the fact that friction cannot be avoided. There is not much chance to study transient processes and most of the attention therefore is focused on steady state solutions.

Computer simulations offer a perfect chance to overcome this limitation as has been shown by the program "**Physics of Oscillations**" developed by Butikov [3].

The program allows in a perfect way to study a broad variety of situations (Figure 7) of an oscillating device, including parametric resonance of non-linear systems.

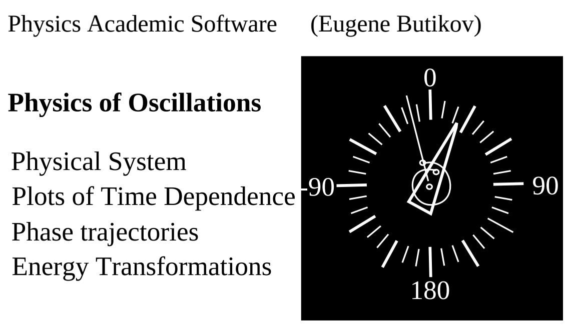


Figure 7 Simulation of a physical system to demonstrate forced oscillations

This study can be carried out without knowledge of any advanced mathematical formalism.

Wave transmission in one dimension

The program TeEl is based on the numerical solution of the telegraph equations. It allows to study transmission processes on a linear system in time domain.

The program offers the possibility to edit voltage and current independently along the line. A simple voltage/ current pulse (Figure 8, right side) can be separated in two parts, a voltage pulse (left side) and a current pulse (middle) and the behaviour of these two parts and the complete pulse can be studied.

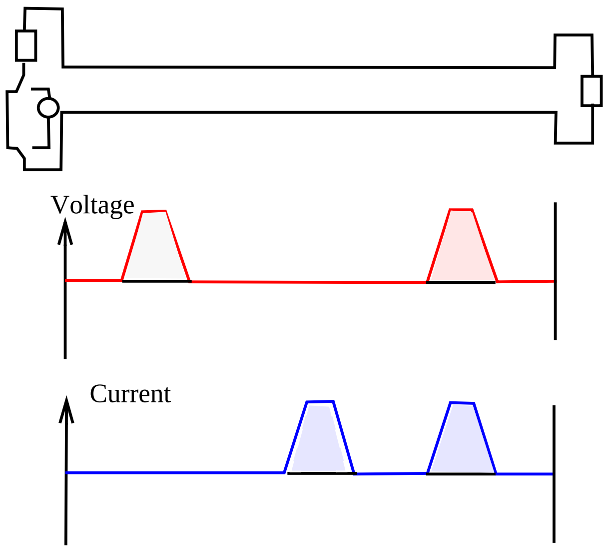


Figure 8 Simulation program TeEl, where pulses of voltage and current can be edited along a double line

Both of these two separated parts, the voltage pulse and the current pulse, are spreading out to both sides (Figure 9 left and middle part). The complete pulse is just displaced to the right. And this can be explained as superposition of the two parts. If the ratio of voltage and current corresponds to the impedance of the line, a very symmetric picture evolves. Both pulses cancel each other on the side opposite to the current and superimpose on the other. The sum is equal to a simple displacement of the original pulse.

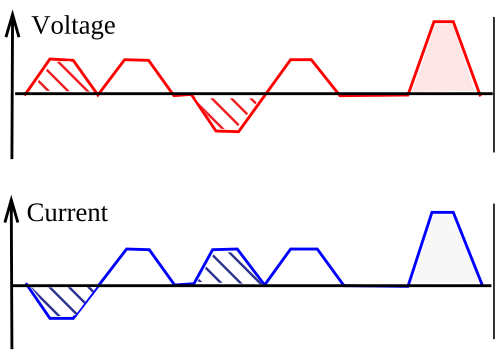


Figure 9 Simulation of the displacement of different pulses on a double line. The sum of the two parts (left side) add up to a simple displacement of the complete pulse (right side).

The parameters of the line may change due to a change in resistivity or impedance, resulting in a change of the ratio of voltage and current. As a consequence, the symmetry of Figure 9 does not hold any more and a reflected pulse will show up.

A research study is planned to find out, how efficient this approach is in comparison with learning results received by traditional teaching methods.

3. Discussion

The common approach of all three examples should have become clear. Interactive simulations are used to support conceptual understanding of the domain under study. The traditional mathematical methods are seen as important and useful but not in a didactical sense. Mathematics can facilitate the work, it can give directions for possible solutions. However, it is not a powerful tool for explaining physics. It should therefore be moved towards the end of the learning cycle so that it does not act as a learning barrier for newcomers.

4. References

[1] H. Härtel, xyZET: A Simulation Program for Physics Teaching. Journal of Science Education and Technology, 9, 3, 275-286, 2000

[2] H. Härtel, The Falling Stick with a > g, The Physics Teacher, 38, 54 - 55, 2000

[3] Eugene I. Butikov, Physics of Oscillations, http://www.msmedia.com.au/phyud.htm