Challenging Faraday's flux law and the Lorentz force
by some simple new measurements on a Faraday disk?

Hermann Haertel
Guest scientist at
ITAP –Institute for Theoretical Physics and Astrophysics
University Kiel

Summary

The question of whether Faraday's flux law is universal or whether there are exceptions has long been controversial. This discussion seemed to have recently come to a conclusion in favour of the generality of Faraday´s Flux Law.
The present article raises this question again with the aid of some rather simple measurements carried out on a Faraday disk. The collected results are surprising and call for an attempt to reconcile them with the supposedly generally applicable Faraday´s flux law. An alternative theory to this law is indicated.

Keywords: Electromagnetic Induction, Faraday´s flux law, Lorentz force, Weber´s fundamental law of Electrodynamics, Faraday´s generator.

Introduction

The subject "electromagnetic induction" with the two basic laws - "Faraday's flux law" and "Lorentz force" - as every experienced teacher knows - is a difficult topic to teach and to understand. On the teaching side, there is the content in textbooks about this topic which is often criticised. Is the notion of a moving magnetic field acceptable or do we have to refer to special relativity whenever a moving magnet is on focus? When treating the Lorentz force as $F= q(E + (v_xB))$, is v relative to the field, relative to the laboratory or to the observer? [Assis, Peixoto, 1992]

On the comprehension side: What kind of mechanism could explain - if only by analogy - how the relationship between a time change of the magnetic flux with respect to an area and the occurrence of a ring-shaped electric field around this area is established? Likewise, the occurrence of a single force without a direct reaction force, acting on a charge carrier moving across a magnetic field, is a single action without reference to any other known process that the students could build on.

In his lectures Feynman comes to the following conclusion: >>We know of no other place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of two different phenomena. Usually such a beautiful generalization is found to stem from a single deep underlying principle. Nevertheless, in this case there does not appear to be any such profound implication. We have to understand the "rule" as the combined effects of two quite separate phenomena.<<
In 1905 Einstein began this paper about Special Relativity with the following paragraph:
>>It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic

action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise—assuming equality of relative motion in the two cases discussed—to electric currents of the same path and intensity as those produced by the electric forces in the former case.<<

Zuza et al. (2016) showed that even rather good students after completion of a full course did not do well when asked to apply Faraday´s flux law or the Lorentz force to experimental setups not yet explained to them before.

On the side of the scientific community there are quite a few basic questions where people disagree. When a magnet is rotating about its polar axes, does the magnetic field, constant in strength everywhere, rotate with the magnet or does it remain stationary regardless of the rotation of the magnet?

Kelly [1998] published measurements that, he argued, confirm a magnetic field rotating together with a rotating magnet. Leus and Taylor [2011] draw the same conclusion, based on their own measurements. By contrast, Chen et. al. [2017] based on their own measurements confirmed that the magnetic field does not rotate with the magnet but remains stationary. Assis and Thober [1994] took a completely different approach and choose the theory of Weber as the basis to explain unipolar induction.

Are there exceptions to Faradays flux law or is this law valid under all circumstances? Feynman [1969] whose arguments have acquired reputation, has stated, that there are situations with change of flux and no induction and vice versa.

Galili and Kaplan (1997) stated that it can sometimes be problematic to use Faraday flux law in its integral form $\varepsilon = \frac{d\Phi_B}{dt} \qquad \Phi_B = \iint_A B \cdot dA$.

As an example, they point to Faraday´s disc, which rotates in a constant uniform magnetic field.

Such statements are criticized by different authors [Scanlon et al, 1969], [Munley, 2004], [Zengel 2019]. They argue that the origin of the exceptions can be traced to an inappropriate choice of the path of the current.

Scanlon argues that ‘such a conflict can truly never arise because of the mathematical identity between $\varepsilon = \oint(E + v \times B)$ and $\varepsilon = -\frac{d\theta}{dt}$ for a given contour‘.

Munley shows in great detail where Feynman fails and states that Faraday's law, properly applied, can be used to calculate the induced emf in any situation where the Lorentz force can be used.

Zengel choose a rather simple model of a Faraday disc to show how the path of the induced current has to be chosen and how Faraday's law has to be applied to show its generality. For this purpose, while determining an area A as well as a change of magnetic flux $d\Phi/dt$ in relation to this area, it is necessary to take into account that the electrons of the induced current are carried along within a moving body and thereby sweep over a certain area. If this area is taken into account, the general validity of Faraday's flux law could be mathematically proven, possibly by performing a transformation to a rotating non-inertial frame of reference.

Zengel's article culminates in the following sentence: "...but neither law should ever predict a result that is inconsistent with the flux rule: $\varepsilon = -\frac{d}{dt}(\iint(B \cdot dA))$"

This sounds as if the discussion about the universality of Faraday law has finally come to a generally accepted solution.

In this paper some rather simple measurements are presented which indicate that there actually could exist such a beautiful generalization and single deep underlying principle which Feynman was looking for.

These measurements are carried out on a so-called Faraday disk as shown on Fig. 1. In 1832, Faraday detected that a rotating magnet together with a rotating disk could function as a DC-generator.

Fig. 1a illustrates such a generator, consisting of a magnet and a separate metallic disc, both free to rotate about the polar axis of the magnet, either independently or anchored together.

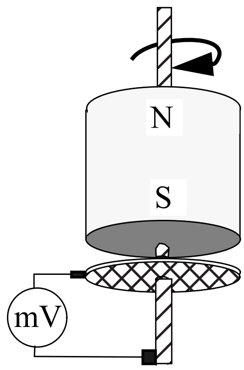


Fig.1 Measurements on a so-called Faraday disk.

Later Faraday found that a rotating magnet, consisting of conductive material could produce an induced dc-current.

The following questions arise when dealing with rotating magnets, which puzzled already Faraday:

1. Why do you observe induction (as expected) with a rotating disk and a stationary magnet, but not with the reciprocal process: a rotating magnet and a stationary disk?

2.Why is induction observed when the magnet and disc rotate together?

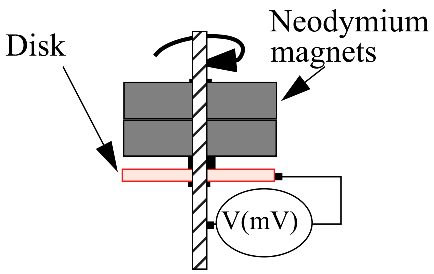
These questions have prompted lengthy discussions. The story began with Faraday's discovery of these paradoxical results in 1832; discussions continue to this day. The key questions in the discussion are those already mentioned above: Is the magnet field of a rotating magnet stationary or does it rotate with the magnet? Is Faraday´s flux law universal or not?

The following measurement are carried out on a system where magnet and a metallic disc are rotating in common. To explain the origin of the induced current in this case, one normally assumes, based on the classical theory, that the magnetic field remains stationary and is not influenced by the rotation of the magnet.

The free conductive electrons inside the metallic disk that is rotating through the stationary magnet field of the co-rotating magnet are cutting magnetic field lines and are accelerated, depending on the orientation of the magnetic field, either towards the rotational axes or towards the rim of the disk. The measured results reported in this paper challenge the idea of a stationary magnet field and the idea of cutting field line as cause of induction.

The Measurements

When starting with measurements on a Faraday disk, a first control measurement showed the linear dependence of the measured induced voltage on the rotational velocity (Fig. 2).



ω(rpm)	920	1280	1750	2620
V(mV)	1,3	1,7	2,4	3,5

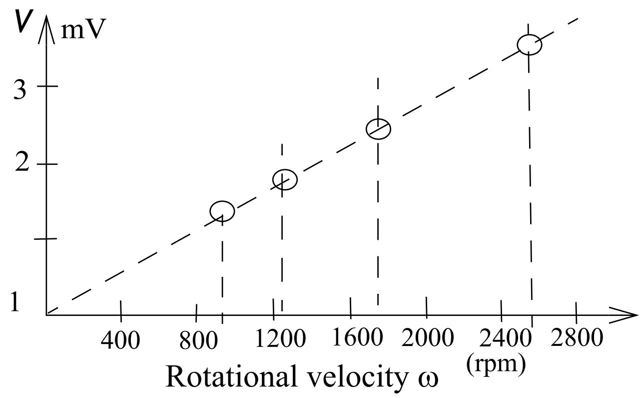


Fig.2 Induced voltage V proportional to the rotational velocity ω.

While carrying out some further measurements and using thin sliding contacts (thin compared to the thickness of the rotating disk), it was observed that the readings were clearly influenced by the position of these sliding contacts on the rim of the rotating disk.

This surprising effect called for a more detailed examination and led to the following change in the experimental setup (Fig. 3):

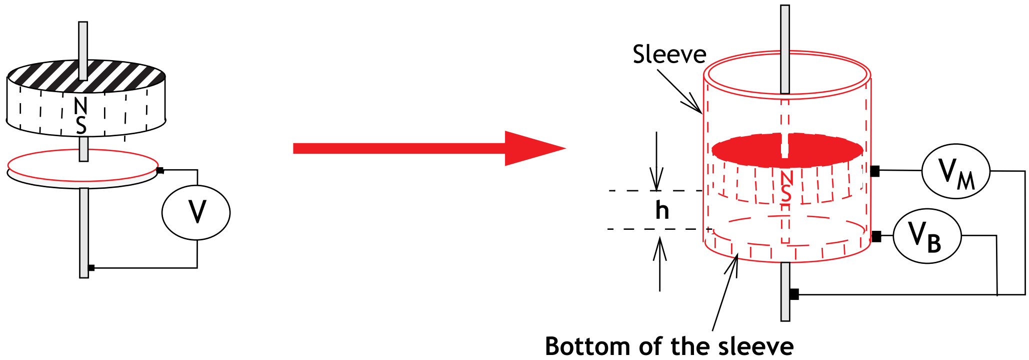


Fig.3 Setup to measure the induced voltage on a Faraday disk, where disk and magnet rotate together around the same axis. The shape of the disc has been expanded to a sleeve, in which the magnets can be placed at different distances from the bottom of the sleeve. Springy wires served as sliding contacts for a connection between the conductors of the external circuit and the rotating parts.

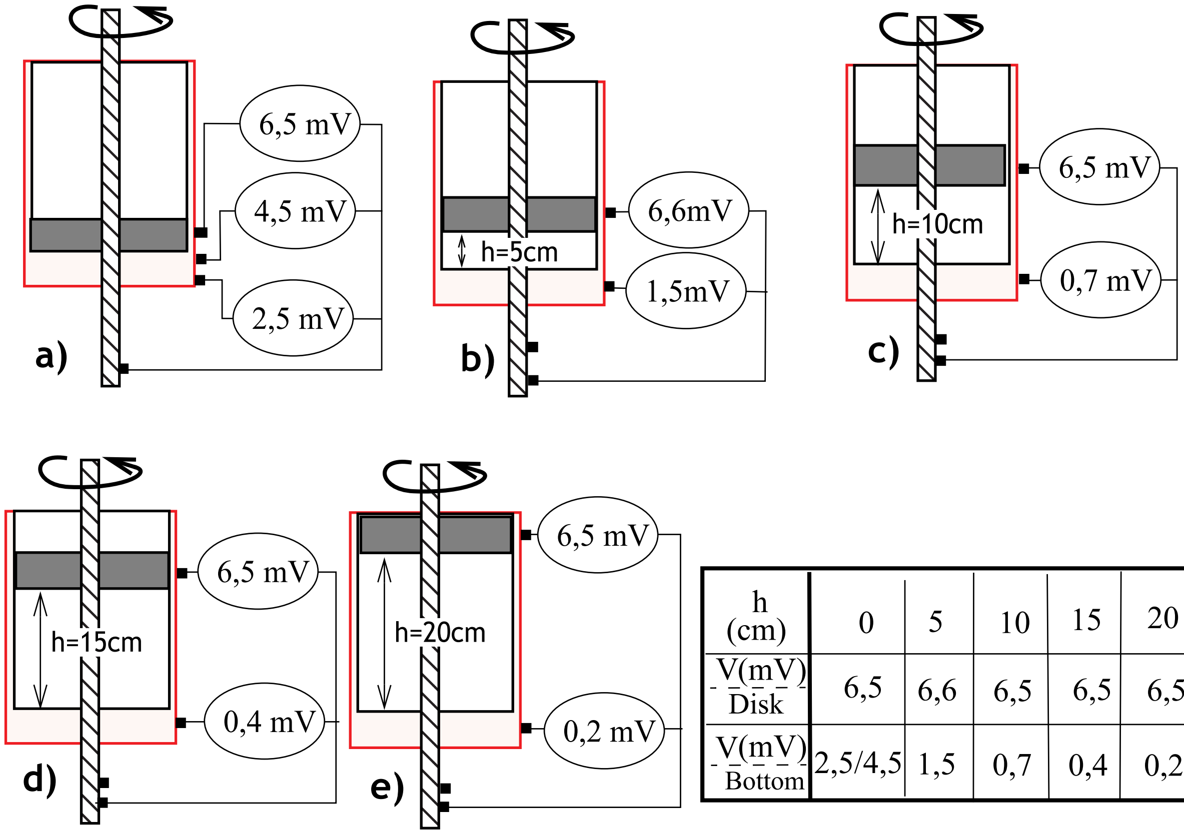
The novelty of this experiment is that the Faraday disk was exchanged by a sleeve where the magnets could be inserted and placed at different position relative to the bottom of the sleeve.

The side walls of the rotating sleeve move in a first approximation parallel to the magnetic field lines. From a classical point of view, this rotating sleeve is therefore not expected to have any major influence on the measured results.

Moreover, one could expect the same potential at the rim of the bottom of the sleeve existing all along the metallic wall of the sleeve.

Finally one can expect smaller readings at the bottom of the sleeve with magnets at larger distance.

This latter expectation, but only this one, is confirmed by the measurements (Fig.4).



h (cm)	0	5	10	15	20
V(mV) Disk	6,5	6,6	6,5	6,5	6,5
V(mV) Bottom	2,5/4,5	1,5	0,7	0,4	0,2

Fig.4 Display of measurements on a sleeve, rotating together with single Neodymium magnets placed at different positions inside the sleeve

Discussion

There are a few unexpected results.

Unexpected is the strong dependence of the readings when taken at different positions at the bottom of the sleeve while a magnet is placed inside directly on this bottom (Fig.4a).

Even more unexpected is the fact that the same induced voltage is measured at the same position as the geometrical centre of the magnets inside the sleeve (Fig. 4 b -e).

A further astonishing results at first sight is the fact that all the different readings do not change if the magnets inside the sleeve are either isolated from the axis and the side walls or brought in good electrical contact with these parts.

This latter result proofs that there are no currents flowing through the magnet but that all measurements concern only the metallic sleeve, the rotation axis and the cables connecting the sliding contacts to the measuring device.

The idea that an induced voltage is caused by cutting of stationary magnetic field lines (and finally by the Lorentz force) is in obvious contrast to these results.

To save this idea, one could try to find good arguments for magnetic field lines being cut by the side wall of the sleeve. A closer look, however, shows that the direction of the magnetic field is either parallel to the side walls or has the opposite directions above and below the position of the magnet in relation to the side wall. It seems not possible to take these facts as good arguments that lead to the observed results.

A more radical conceptual change in order to save the idea of cut magnetic field lines as the cause of induced voltages is the assumption that the magnetic field is rotating with the magnet. Under this assumption the rotating field lines would be cutting the external part of the electric circuit to cause the induced voltage.
There are quite a few arguments why it is problematic to talk about a moving magnetic field. How could its velocity be defined . What would be the result of a Lorentz transformation according to Special Relativity.
In addition to the concept of a rotating magnetic field, an argument would have to be found, why the polarisation on the side wall of the sleeve (the cause of the observed induced voltage) has its distinct and sharp maximum always exactly opposite to the geometrical centre of the magnet.
According to Zengel, it should always be possible to find an area, either directly or inside of moving parts which are swept over by those electrons that are part of the induced current and thus are defining an area to correctly apply Faradays flux law. Such electrons do not seem to exist.
With an unbiased look at the measured results achieved here, it looks as if it is the distance between the sliding contact and the magnet that controls the induced voltage. It looks as if the conduction electrons inside the sliding contacts are interacting with the electric charges that are causing the magnetism of the magnet and that this interaction is causing a polarisation of the external circuit.
A theory that is proposing such an interaction as the cause of any induced voltage has been published in the 19th century by Wilhelm Weber. Here is not the place to explain this theory in any detail. A short description will be given in the appendix.
First the reported measurements in this paper should be made public and should be discussed in the light of the established theories: Faradays flux law and the Lorentz force.
Is there any possibility for an interpretation of the presented measurements that is in agreement with these classical laws? At the end of such a discussion, if negative, a serious look at Webers theory could possibly offer a fruitful answer. [Weber, 1846], [Assis, 1994]

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Appendix

In 1846 Wilhelm Weber presented his force law (Weber, 1846). The starting point for him was Faraday's flux law and Ampère's law (the original one, that is, Ampère's

central force between two current elements satisfying Newton's action and reaction). At that time the laws of Ampère and Faraday were published as unrelated and independent laws. Weber suspected that they had to be based on a common fundamental law of electrodynamics.
Based on quite sophisticated measurements and being not only a great experimenter, but also an equally great theoretician (he collaborated with Gauss), he succeeded in deriving the presumed fundamental law from his measurements.
This law is an extension of Coulomb´s Law. This means first of all, that as in electrostatics the Newtonian action/reaction principle applies in its strict form: the forces between interacting particles are not only of equal size, but act exclusively in the direction of the interacting partners.

New are two additional terms, the first contains the factor $-v^2/c^2$, the second the factor $+a/c^2$.
Weber's Fundamental Law describes the mutual force $F_{1\to 2}$ and $F_{2\to 1}$ between two charge carriers q_1 and q_2 at their mutual distance r_{12} and reads as follows (in modern notation):

$$\vec{F}_{1>2} = \frac{q_1 q_2 r_{12}^0}{4\pi\varepsilon_0 r_{12}^2}\left(1 - \frac{v^2{}_{12}}{2c^2} + \frac{r_{12}\cdot a_{12}}{c^2}\right) = -\vec{F}_{2>1}$$

$F_{1\to 2}$ means the force from particle 1 acting on particle 2 and accordingly for $F_{2\to 1}$.
The terms v_{12} (dr/dt) and a_{12} (d^2r/dt^2) denote the relative velocity and the relative acceleration between the interacting partners. The term $r^0{}_{12}$ denotes the unit vector along the interacting partners. The constant c, first introduced by Weber, was later experimentally determined by him together with Kohlrausch as being identical in physical dimension and size with the speed of light (Weber, Kohlrausch, 1857).
Weber's force is compatible with Ampère´s law [Assis, 1990] and compatible with the conversation of energy. [Weber, 1872]. It is open for the extension toward retardation potentials.
To apply this law to the case of a rotating magnet and a disk (in our case a sleeve) one has to look for subsystems with relative velocities and or accelerations. In this case there are on one side positive and negative charge carriers forming the external circuit (at rest relative to the laboratory) and on the other side all atoms and electrons forming the magnet, rotating together with the sleeve. Most of the interactions between these subsystems cancel because there are an equal number of positive and negative charge carriers involved. An asymmetry exist, however, on the one side between the conduction electrons within the external circuit, being not fixed but free to be moved (at least to a certain extend) and on the other side those charge carriers that are causing the magnetism of the magnet and are moving relative to the laboratory. In a first approach one can think of surface currents analogous to a current in a coil producing the same magnetic field.

These charge carriers, responsible for the magnetism will interact with the conduction electrons in the external circuit and will lead to a polarisation of the latter. Since the interaction in Weber´s equation is strongly dependent on distance the position of the sliding contact together with the thickness of the sleeve (its side wall) will have a dominant influence on the size of the induced voltage.