

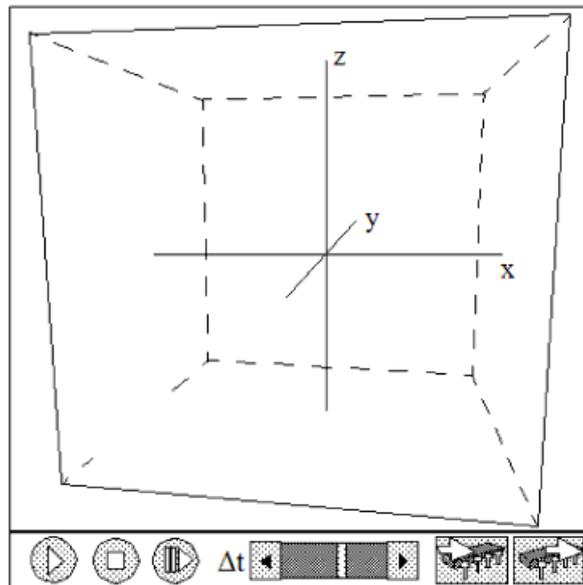
Introduction to Classical Mechanics

Computer Supported Materials for Selected Topics

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1. Introduction

1.1. Contents

The following learning material is an advancement from a former package, which was based on the simulation program xyZET. This program allowed in a three-dimensional representation to animate particles under the influence of various forces (elastic, electric, magnetic, gravitational, contact).

The program xyZET was written in C under UNIX and therefore needed an x-server to run under Windows. To avoid this obstacle for wider distribution and to become platform independent a reduced version of xyZET has been re-written in Java. and is called JavaXYZ (Programmer: Sasa Divjak). JavaXYZ has the form of a workbench and offers the possibility to develop simulations of a broad variety without any knowledge in programming. Only mouse controls and a knowledge about the interface are necessary.

In addition to the prepared simulations and corresponding text a series of computer animated videos have been developed to reduce the gap between a real experiment and the corresponding simulation.

From the traditional curriculum those topics were selected where interactive simulations can play a supportive role for motivation and understanding. For more theoretical topics the students have to refer to the corresponding textbooks.

List of selected topics

Velocity and Acceleration, Force, Falling Objects, Collision, Circulating Objects, Oscillating motion.

1.2. Treatment of Mathematical Aspects for Teaching Physics

The material presented here is based on an alternative approach in comparison to traditional teaching. A main aspect of this new approach is the attempt to avoid relying on mathematical knowledge as a prerequisite for understanding physical concepts. Instead, interactive simulations in combination with animated computer graphics are used to promote the integration of mathematics and physics.

Following this principle, mathematical equations are not derived from theory when introduced but directly presented. By comparing these mathematical expressions with the results presented by the simulation the students shall get acquainted with these results.

1.3. Integration of Simulation and Experiment

When using the materials presented here, it is taken for granted that real experiments are demonstrated and carried out in class whenever feasible. It is a rather trivial fact that a simulation cannot prove anything but needs the proof and support of the real experiment. The teacher has to point out this fact to ensure that students do not stick to any naive and non-reflected opinions about the value and significance of simulations compared to actual experiments.

1.4. Measurements and Units

When measurement are taken while real experiment are performed, it is vitally necessary to note the actual value as well as the used units.

When using a simulation with output in numerical form, the choice of the units is generally free. For the program JavaXYZ for instance, the defined unit for distance corresponds to

something like 1/10 mm, depending on the available resolution of the monitor. This unit can represent any distance in real life. The same is true for other numerical values of time, mass, charge etc.

Before using such numerical values presented by a simulation, an agreement has to be found concerning the units to be used. It is advised that the user chooses the common basic units like m, sec, kg, and the derived units like N or m/s, in compliance with the system of units used by the scientific community.

1.5. Media

Videos

The inserted Videos are either pure computer animations or a combination of real experiments and computer animations.

Authors: Jan Paul, Dug Van Dang

2. Simulation Tool Java-xyZET

2.1. General Information

For the support of the following course material a simulation program is available, which allows a 3d-presentation of all simulations and this inside of a cube, which can be rotated and presented from all perspectives.

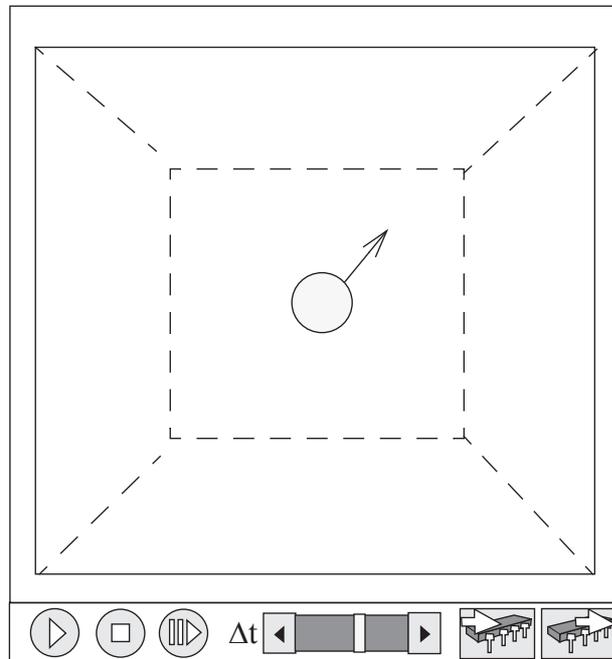


Fig 2.1.: Main simulation window with control buttons

This program can be downloaded at
http://www.astrophysik.uni-kiel.de/~hhaertel/index_e.htm.

2.2. User Interface

The cube and its content can be rotated by clicking with the left mouse button down within the cube and moving the mouse.

If in addition the shift button is down, the cube can be moved. With the ctrl-button down the cube can be zoomed in and out.

The buttons   allow to start or stop the simulation. To advance the simulation by single step the step button  has to be activated.

The slider of Δt allows to change the speed, with which the simulation is displayed.

Activating the save button  will keep a copy of the actual state of the simulation in memory, which can be recalled to the screen at any time by activating the recall button



2.3. Right-angled Coordinate System

Sequence of the axes

Within the cube of Java-XYZ a right-angled coordinate system is used to define positions and directions.

By convention the sequence of the axes for such a right-angled coordinate system has been determined by the so-called "right-hand-rule". The thumb points in the direction of x , the forefinger points in the direction of y , and the middle finger in the direction of z .

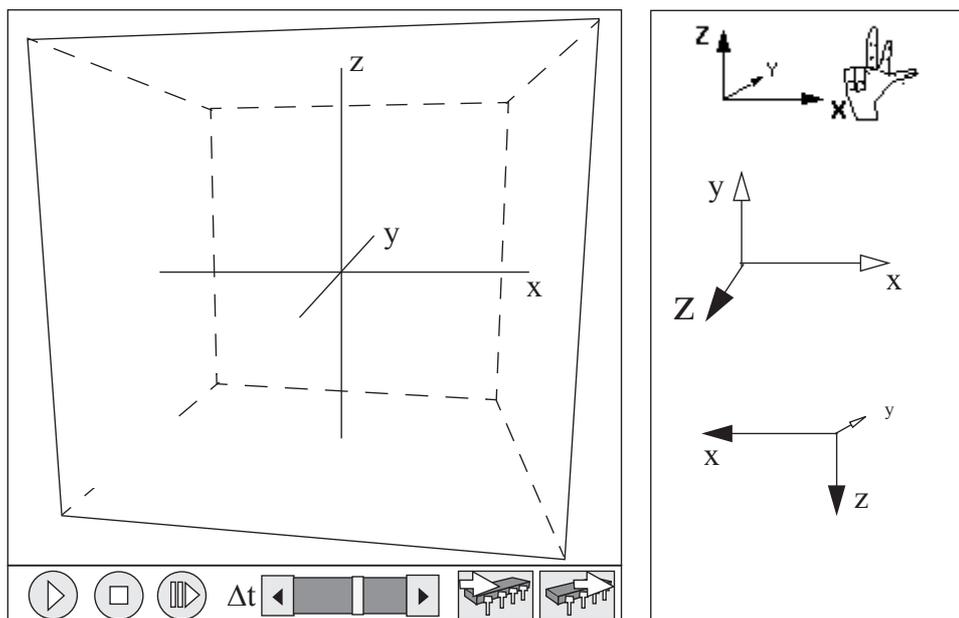


Fig 2.2.: xyz-axis under different perspectives

As long as this sequence is respected, any orientation in space is equivalent as demonstrated with the three possibilities to the right.

As default view within Java-xyz the position shown above has been selected. In this default position the positive x -orientation points to the right, the positive y -orientation points to the back, and the positive z -orientation points upwards.

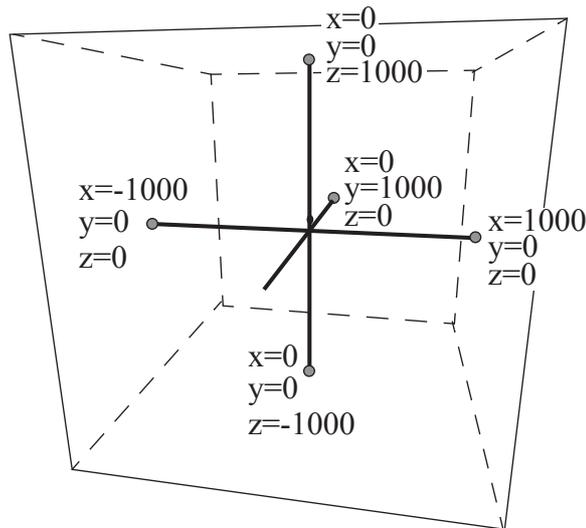
2.4. Coordinates in Space

Units of length

The length of the three axes x , y , and z from the origin to the walls of the JavaXYZ cube is divided in 1000 units. The length of each edge of the cube is therefore equal to 2000 units

The actual length of this unit on the screen corresponds to about 0.1 mm. However, since you are dealing with a simulation there is no reason, not to think of such a unit as representing a larger length, for instance 1mm. In this case the JavaXYZ cube would represent a cubic volume with an edge length of 2m.

Positions



Based on a system of rectangular coordinate axes, each position in space can be identified by three numbers, called coordinates. To do so, it is necessary to define the positive orientations of x , y and z by following the right-hand-rule described above.

A specific position within this cube is described by three numbers, the x -, y - and z -coordinates in this sequence.

Example: $P(1000, -1000, 1000)$ indicates the upper right corner of the front side of the cube. To get used to this structure and its different coordinates it may be useful invest some time for exercising.

Fig 2.3.: XYZ-Cube with axis and coordinates

Below you find a list of positions for particles within the XYZ-cube. In may be useful to determine in thought where these particles should be located before they are placed?

$P(1000, 0, 0)$; $P(1000; 1000; 1000)$; $P(-1000; 0; 0)$; $P(-500; -500; -500)$;

$P(+500; +500; +500)$

To place a particle at a wanted position there are two possibilities: You can either use the „Particle Management“ window (at A in fig.2.4) where you can place a particle at a predefined location (at B and C).

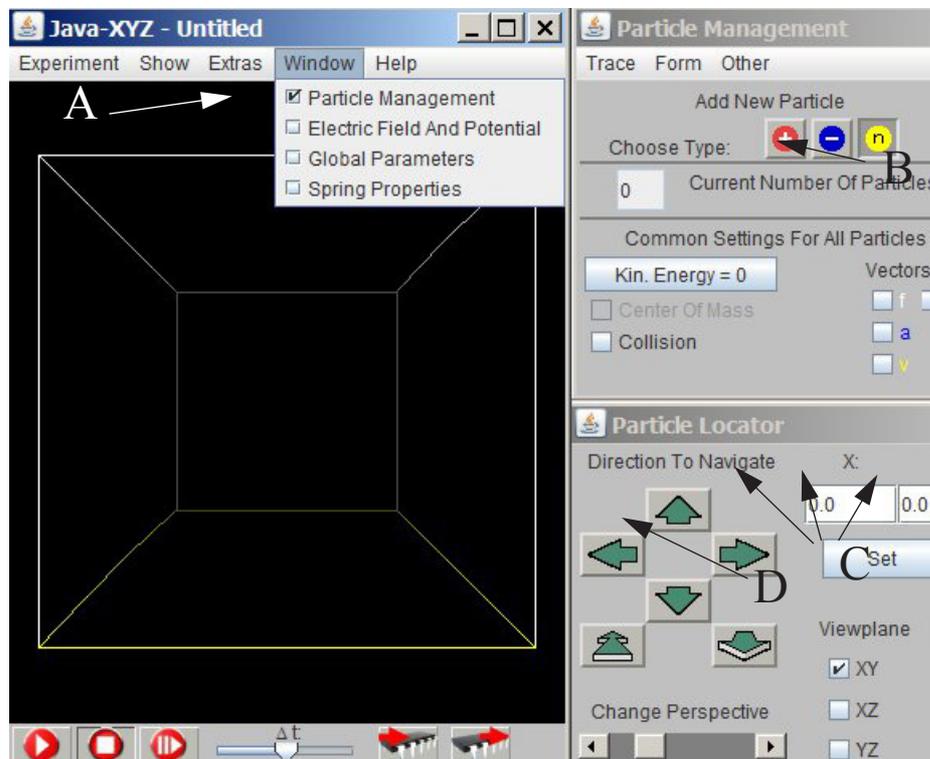


Fig 2.4.: To set a particle, the Particle Management window has to be selected (A) The type of the particle can be selected at B. Its position can be determined either by entering numbers at C or by activating the green arrows (D).

The second possibility is to open the "Particles Inspector" window of a particle (which should have been placed before) by selected the particle with the **right** mouse button (see fig.2.5) and then the „Edit Particle“ submenu. The "Particles Inspector“ window offers the possibility to enter or change the coordinates numerically. When doing so each individual input has to be finished by hitting the “enter“ key

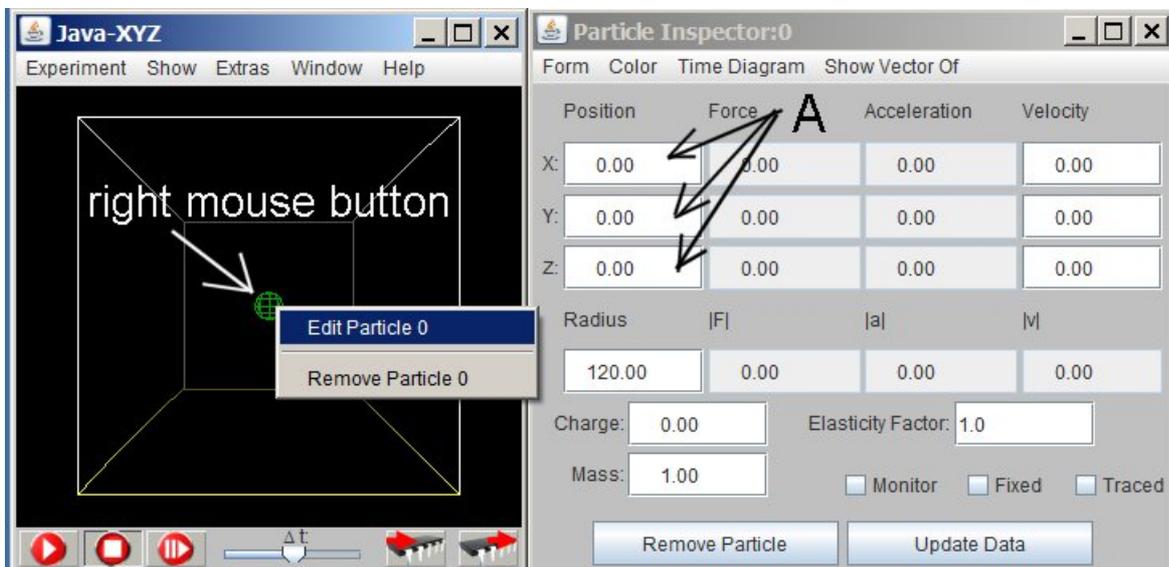


Fig 2.5.: XYZ-cube with the Particle Inspector window where the position of the particle can be entered numerically (A)

3. Velocity and Acceleration

3.1. How to Add and Subtract Velocities

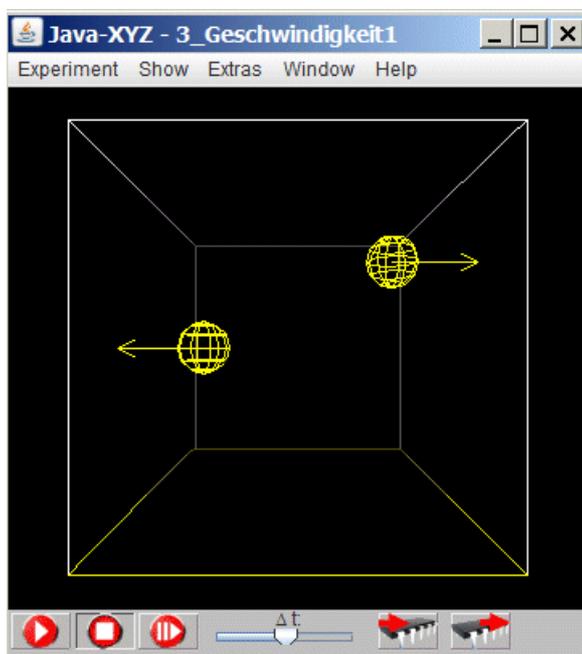
Some terms in physics like velocities, forces, a.o. have two distinct properties: an amount and a direction. Such terms are called vectors.

If a velocity is bound to a specific direction, it can have 2 orientations, either to the left or to the right, up or down, north or south a.o.

In physics and mathematics such two opposite orientations are by convention symbolised as positive (+) and negative (-). These symbols - even though they look alike - are not identical with the mathematical operations addition (+) and subtraction (-).

If velocities with positive or negative orientations have to be added or subtracted the following rules apply:

To add a velocity with negative orientation $+ (-)v$ is the same as to subtract velocity with positive orientation. In mathematical symbols: $+ (-)v = - (+)v$



To subtract a velocity with negative orientation $- (-)v$ is the same as to add the same velocity with positive orientation. In mathematical symbols: $- (-)v = + (+)v$

The rules to be applied are the same as adding or subtracting positive or negative numbers.

If a velocity is symbolised as $(+)v$, it should be called positively oriented velocity while the short form "positive velocity" can lead to confusion among newcomers to the field. There is no positive or negative velocity, there are only positively or negatively oriented velocities and this only relative to a specific direction in space.

What has to be done to increase the speed of both particles in simulation 3_velocity".

Fig 3.1.: Simulation "3_velocity"

The following picture illustrates, how to detect or change the velocity (its amount) of a selected particle.

This can be done by entering the numerical values for the three components v_x , v_y and v_z in the "Particle Inspector" window at A

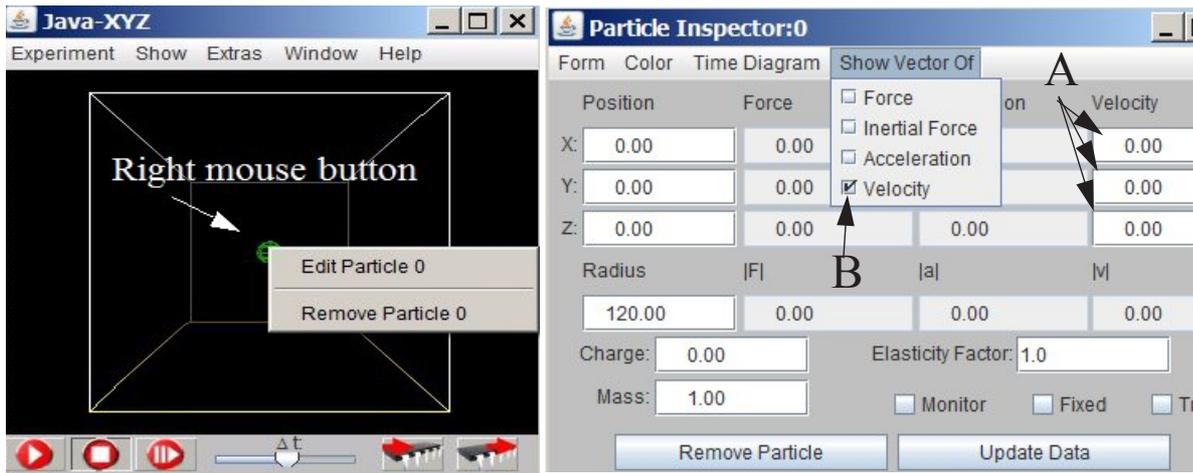


Fig 3.2.: How to change the velocity and how to activate the visualization of vectors

The velocity of a particle can be visualised as arrow - the symbol for a vector. This functionality can be initiated in the "Particle Inspector" window at B (fig. 3.2.).

The visualisation of v common for all particles within the XYZ-cube can be set in the "Particle Management" window at C (fig. 3.3.).

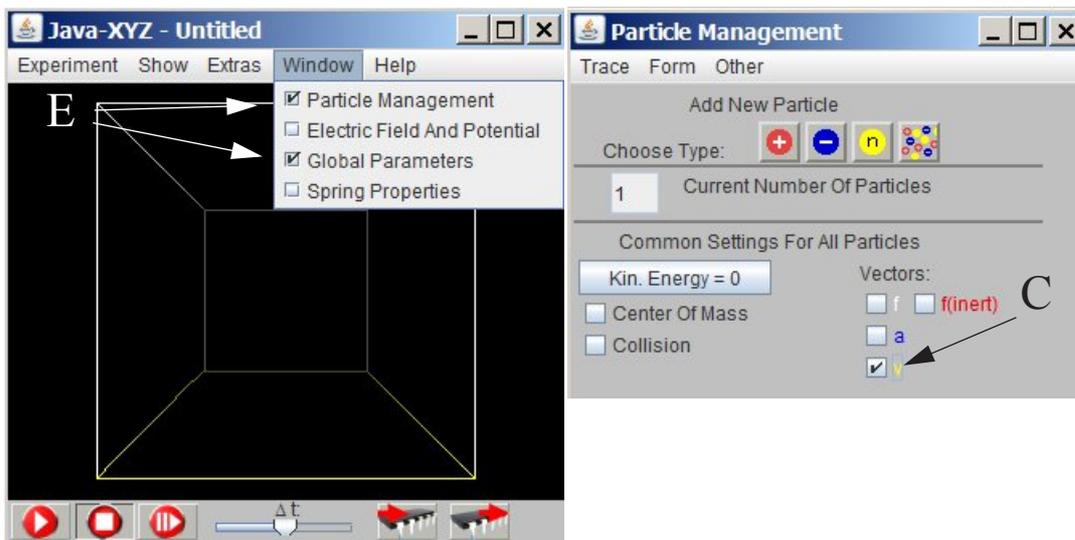
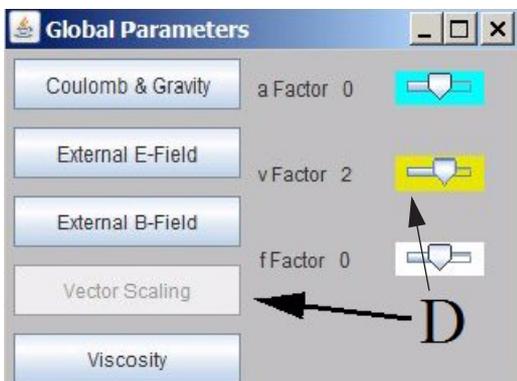


Fig 3.3.: How to activate the visualization of vectors common for all particles



The scale of the visualized vectors (for acceleration, force and velocity) can be set in the "Global Parameter" window at D.

Fig 3.4.: How to scale arrows, the symbol for vector quantities

The last two windows are called from the main menu line of the Java-XYZ window at E (Fig 3.3.at E)

3.2. Velocity in 3 Dimensions

A velocity with an arbitrary direction in space can be decomposed in so-called x-, y- and z-components, directed along the 3 axis of a system of rectangular coordinates.

These components can have 2 orientations along each axis, labelled either as positive or negative.

As an exercise to get familiar with coordinates within the Java-XYZ cube as well as with velocity components, the following 4 similar exercises are proposed where each time the velocity components of the particle in the middle should be set in such a way, that it will collide head-on with the particle in the back. For changing the velocity components of a particle see fig. 3.1.

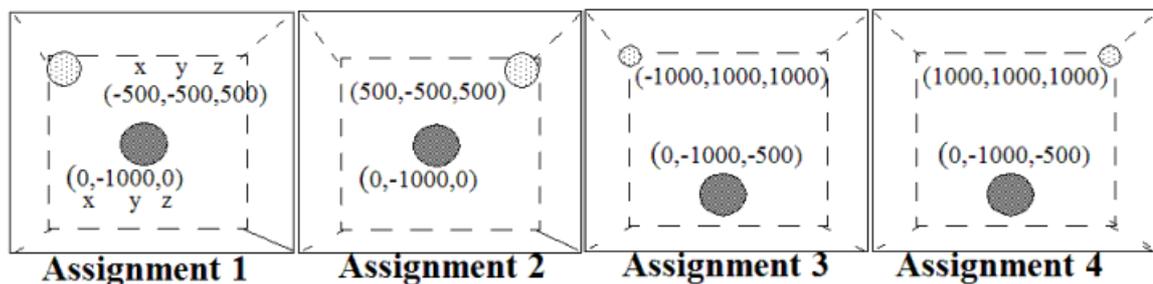


Fig 3.5.: Assignments to exercise with coordinates and v-components in 3 D

A reminder:

The cube can be rotated when moving the mouse with the left mouse button down.

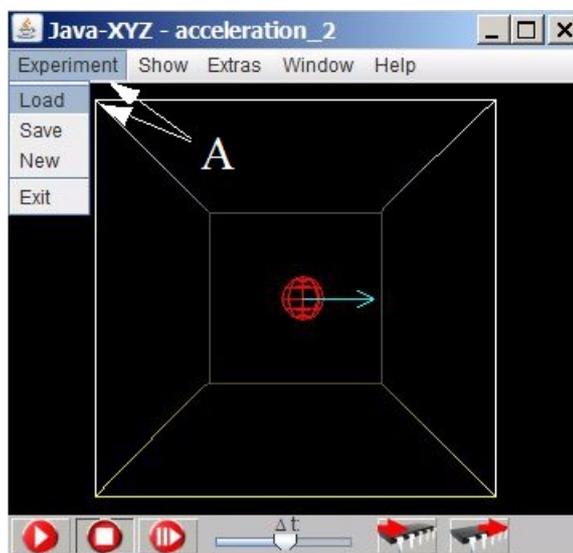


Fig 3.6.: How to load a prepared simulation

You may either set the 2 particles at the given positions yourself or you can load the prepared simulations named:

3_assignment1 till _assignment4.

Prepared assignments or simulations can be loaded under the menu "Experiment" in the main menu line of the XYZ-cube (see fig. 3.6 at A).

3.3. Velocity and System of Reference

Rest and Motion

For our daily life the two states "being at rest" and "being in motion" are clearly distinct. This idea has first been questioned by Galileo and Newton in the 17th. century.

There is no place in the whole cosmos which could be claimed to be at rest. We are placed on a rotating earth, we are circulating around the sun, we are moving with the sun across our galaxy, etc.

What we so distinctly perceive as being at rest, is therefore only relative, only apparent.

To define a place to be at rest or to move with a specific velocity, a system of reference is indispensable.

The importance of such a system of reference becomes quite obvious if the movement of a planet, for instance Mars, has to be described.

Movement of Planet Mars from Geocentric versus Heliocentric Perspective

To understand the retrogradation of our planets, the geocentric and the heliocentric perspective have to be unified in thought. To support this mental task, a video is prepared (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/mars_en.htm) which shows pictures from these two perspectives with smooth transitions.

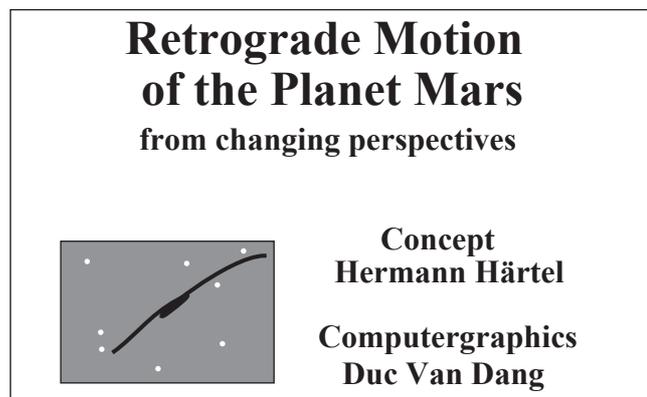


Fig 3.7.: Video about the movement of Mars

To simplify the explanation, it may be useful to rectify the circular movement of Earth and Mars in the following way.

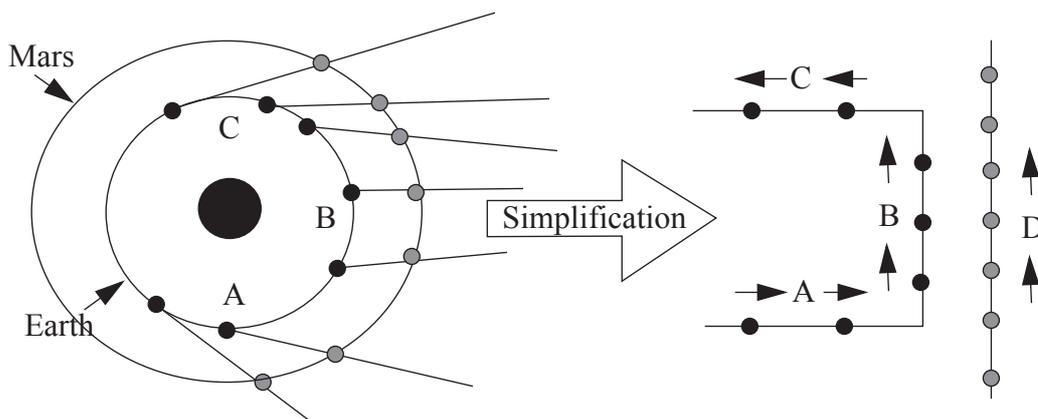


Fig 3.8.: Explanation about the retrograding movement of Mars

The circular path of Earth is simplified to the track A-B-C, while the path of Mars is simplified to a straight line D.

While on track A, it is obvious that Mars seems to move to the left.

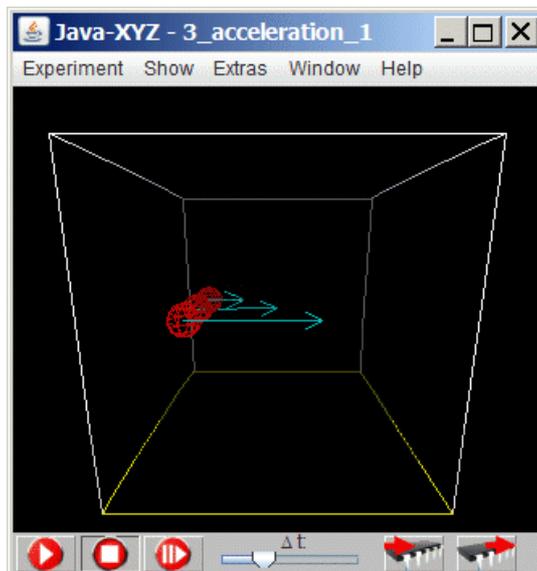
Since the velocity of Earth on track B is faster than the velocity of Mars, it seems that Mars is moving to the right. On track C as on track A it looks again as if Mars is moving to the left.

3.4. Acceleration

In daily life the meaning of "acceleration" is limited to an object moving with increasing speed. The meaning of this term in physics, however, is more general.

When ever the velocity of a moving object is changing, either increasing or decreasing its speed or changing its direction, the corresponding movement in physics is called to be accelerated.

If the velocity of an object is changing from v_1 to v_2 , during a certain time period Δt , in physics the corresponding movement is called to be accelerated.



The acceleration of a moving object is larger, if the difference $v_2 - v_1$ is larger for the same time interval: In mathematical terms this implies: $a \sim v_2 - v_1$

The acceleration is smaller, if the time interval $\Delta t = t_2 - t_1$ is larger to reach the same difference in velocity: In mathematical terms this implies: $a \sim 1/(t_2 - t_1)$

This leads to the mathematical definition of ac-

$$\text{celeration as: } a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Fig 3.9.: Simulation "3-acceleration1"

The particles in the simulation "3_acceleration-1" move with different acceleration (for loading a prepared simulation see fig.3.6). This can be checked by stopping the simulation in the middle and checking the velocity component v_x inside the "Particle Inspector" (fig. 3.2)

Acceleration as Vector

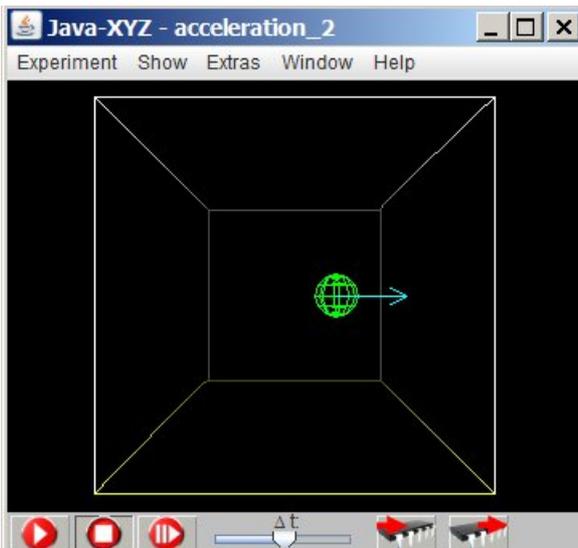
The physics term acceleration has vector properties. This means that it has a direction in space and that it can be decomposed along the axis x, y, and z parallel to the axis of a rectangular coordinate system.

For a given direction the acceleration as vector can have 2 orientations, indicated as positive and negative.

These indications are often identified with acceleration and deceleration.

However, this not correct in the world of physics. A particle which moves with a negatively oriented acceleration does not mean, that the velocity of the particle is decreasing. It may well increase.

The prepared simulation "3_acceleration2" may help to clarify this *issue*.



The blue arrow indicates the actual acceleration. The velocity is permanently changing but the acceleration is always constant and has a positive orientation (parallel to the positive direction of the x-axis).

The acceleration in this simulation is caused by simulating a charged particle and an electric field in +x-direction, causing a force in +x-direction.

*Fig 3.10.: Simulation "acceleration2"
Movement with constant acceleration
but permanently changing velocity*

The orientation and amount of the acceleration can be changed inside the "Particle Inspector" window by changing the amount of the charge (A) or the polarization (plus instead of minus) of the charged particle.

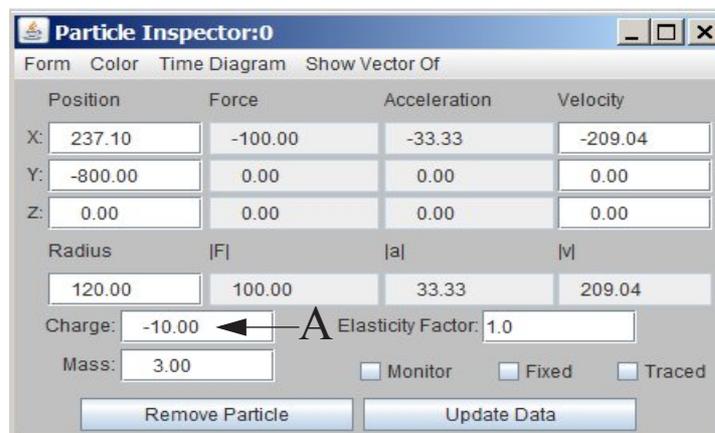


Fig 3.11.: How to change charge and polarity of a selected particle

A further possibility to change the amount of the acceleration (the applied force) is given by changing the strength of the simulated electric field (see fig. 3.12)

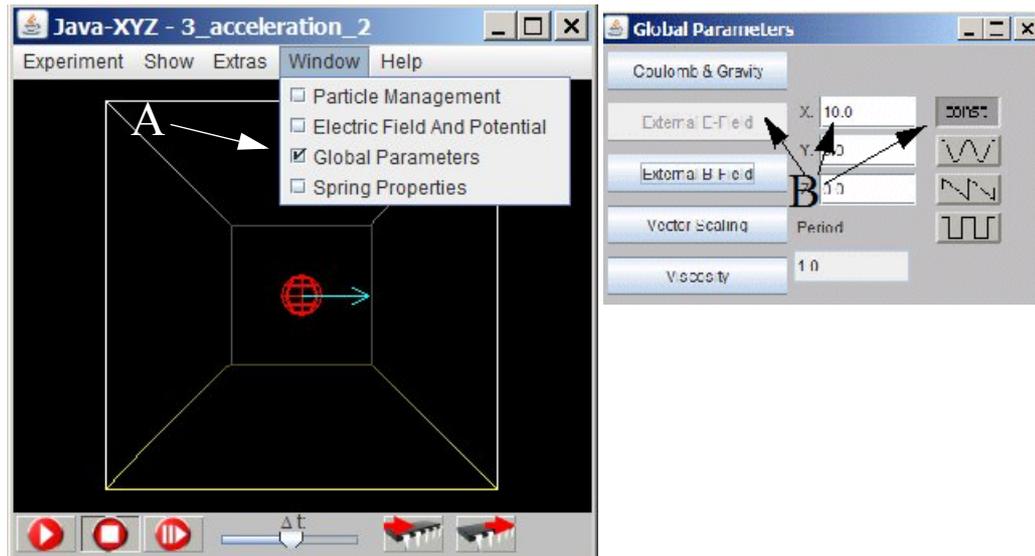


Fig 3.12.: How to set or change the strength of the electric field (at B) inside the "Global Parameters" window, called at A in the Window sub-menu (main menu line of Java-XYZ)

Remark:

The acceleration during the collision with the walls of the cube are not shown.

Result

The orientation of the acceleration - either positively or negatively oriented - has nothing to do with increasing or decreasing speed. With both orientations the object can move faster or slow down.

Important is in what sense the velocity is changing. If the velocity changes to more positive values or towards the positive orientation, the acceleration has a positive orientation and vice versa. In mathematical terms this rule can be expressed as:

acceleration with positive orientation: $(v_2 - v_1 > 0)$,

acceleration with negative orientation: $(v_2 - v_1 < 0)$

4. Concept of Force

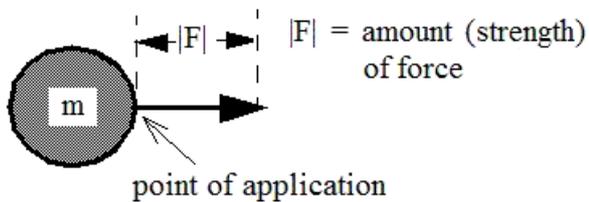
4.1. General Properties

In physics the concept of force is used since Newton to understand and describe mechanical processes.

A so-called Newtonian force as it is understood in physics has the following properties:

- A force is the cause of change, either of the movement of an object or of its form or volume.
- A force is always the result of an interaction between at least two objects. This implies that a force can never exist on its own, it always appears in pairs. These two paired forces have always the same strength and direction but opposite orientation.
- A force has a unique direction with two possible orientations, usually indicated as positive or negative.
- A force has a point of attack or point of application.

4.2. Force as Vector

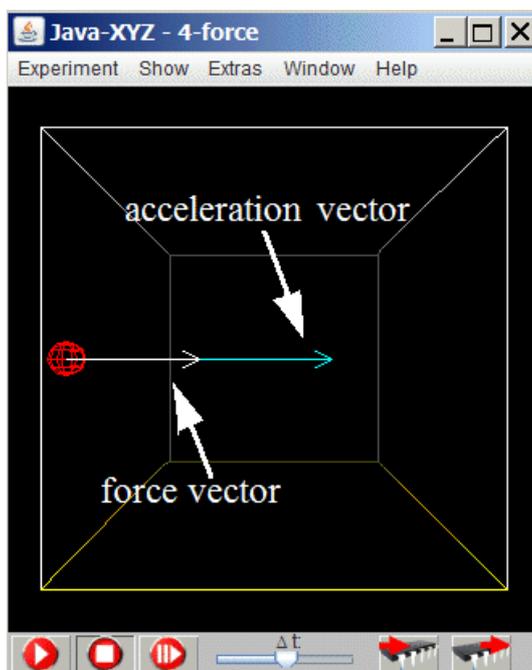


Every force as a term in physics has two different properties: an amount and a direction.

A term with these two properties is called a "vector".

Such a vector is represented and visualized as arrow

4.3. Force and Acceleration



The prepared simulation "4-force" demonstrates the effect of a force on the movement of a particle.

As in the previous simulation the force is simulated by setting the charge of the particle to a positive value (in the Particle Inspector window) and giving the strength of an electric field (in the positive direction of the x-axis) a positive value (in the Global Parameters window).

The arrows indicate the directions and amounts of the applied force f (white) and the resulting acceleration a (blue). The latter two values are indicated numerically in the "Particle Inspector" window.

Fig.4.1.: Simulation "4-force";
Movement with constant acceleration

By changing the mass of the particle (in the "Particle Inspector" window) the simulation shows that the acceleration a and the mass m are inverse proportional. If you double the

mass, the acceleration is divided in halve and vice versa. This relation is valid in general and should be intuitively clear: the more mass an object has, the slower can it be set in motion, assuming a constant force. Such an inverse relation implies, that the product of mass and acceleration is constant.

More to this relation will be found in the next chapter.

4.4. Newtonian Force, Inertia and Mach's Principle

Newton's 2nd Law

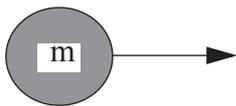
In the former chapter it has been stated, that under the influence of a constant force the product of mass m and acceleration a of a moving object is constant.

This relation - the inverse proportionality of m and a - has been tested experimentally and has been found to be valid under all circumstances, at least unless not very high velocities (comparable with the speed of light) get involved.

This product - $m \times a$ - has been taken as definition and as a measure for the strength of a force which leads to the famous equation, set up in the 17. century by Isaac Newton:

$$F = m \times a$$

- m = mass
- a = acceleration



In practical all physics textbooks, a situation according to this equation is visualized by a single vector F applied to an object with a mass m .

Inertia

From experience in daily life we know that an object resists, if we try to make it move faster or if we want to reduce its speed.

In traditional physics this resistance is called inertia and is seen as a property of matter. And as a property of matter, it cannot be a Newtonian force. An important and indispensable character of a Newtonian force is that it has to occur as interaction between two objects. It cannot be the property of a single object. Furthermore inertia can never accelerate an object. It can only react on acceleration.

And since such an interactive partner is missing, the resistance of a material object against any kind of acceleration is not seen as a force and is not visualized as such in traditional textbooks.

Mach's Principle

There is an exception, which dates back to the beginning of the 20 century. At that time, the German physicist Mach proclaimed some principles about general relativity and published the idea that the presence of the matter of all the galaxies in the universe is responsible for the inertial mass of objects here on earth.

The broad notion is that "mass there influences inertia here".

Until today this idea has never been experimentally approved or disproved and therefore remains speculative. However it opens the possibility to think of a possible interactive partner for inertia and facilitates to accept the resistance of a material object against acceleration as a real force.

In the following learning material the possibility is offered to accept this idea of inertia as a real force and to visualize it, when ever wanted, as follows.

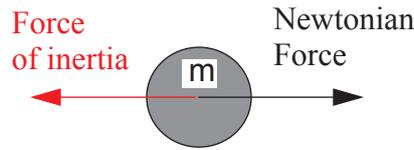


Fig.4.2: Visualization of a Newtonian force and the equal and oppositely oriented force of inertia as an acceleration-reaction-force

By doing so it has to be kept in mind that a force of inertia, even though it is real, is not a Newtonian force, which can be applied to accelerate an object. The force of inertia is an acceleration-reaction force, which only exists during the process of acceleration. It cannot be applied to an object to cause acceleration. But if a Newtonian force is applied to an object this force of inertia makes acceleration possible by producing the necessary resistance.

Without it the Newtonian force would act against something without resistance, it would pull or push against nothing or emptiness. It seems reasonable to claim that this could have no effect.

The following simulations demonstrates different kinds of accelerated movements and allows the user to visualize independently the Newtonian as well as the forces of inertia.

4.5. Force of Inertia

With the idea of inertia as an acceleration-reaction force the dynamic situation, where an object is accelerated by a Newtonian force can be described as an equilibrium between two forces, a Newtonian force and a force of inertia - an acceleration-reaction force.

Both forces can be visualized independently as vectors.

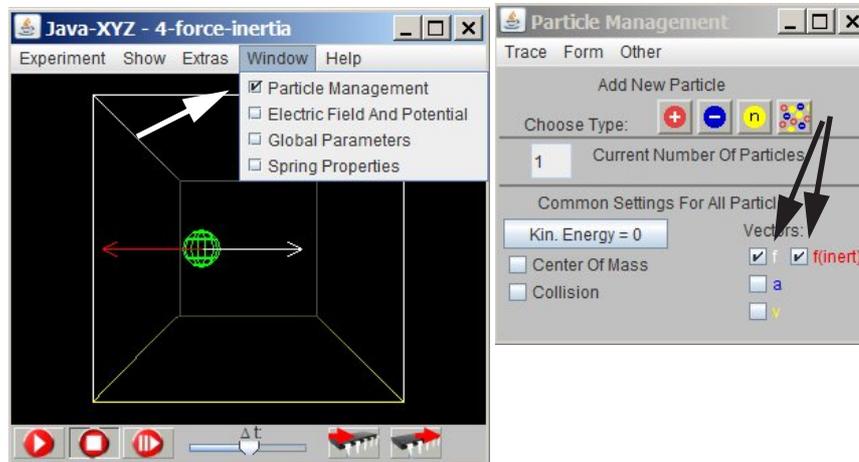


Fig.4.3: Simulation "4-force-inertia". How to visualize a Newtonian and an inertial force (an acceleration-reaction force) common for all particle to be set

The "Particle Inspector" window offers the possibility to visualize these forces for an individual particle.

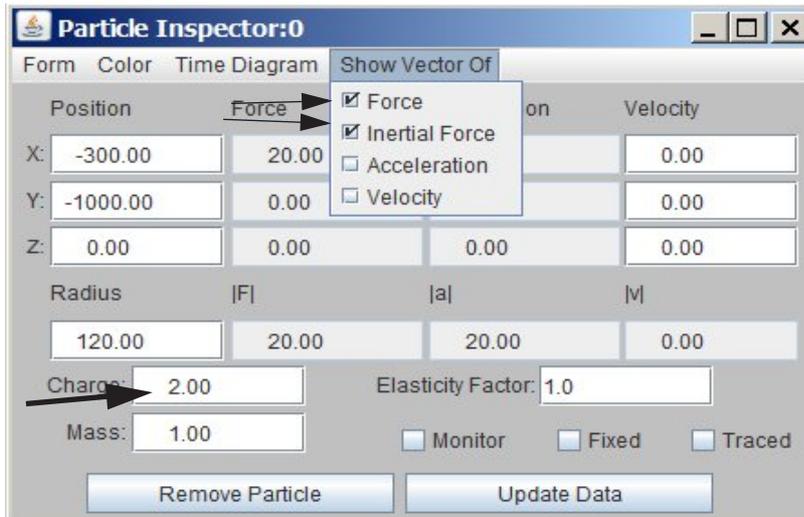


Fig.4.4: How to visualize a Newtonian and an inertial force (an acceleration-reaction force) for a selected particle

As already mentioned above an accelerated movement can be described as if there exists always an equilibrium between an applied Newtonian force and a force of inertia as acceleration-reaction force. This description of a movement is valid for all objects independent of their mass.

The accordance of the simulation with this last statement can be checked by changing either the mass of the particle or its charge (in the "Particle Inspector" window).

Mathematical description

With the idea of inertia as an acceleration-reaction force, the movement of an object being accelerated by an applied Newtonian force can be interpreted as an equilibrium and represented in mathematical form as:

$$F_{\text{Newton}} = - F_{\text{Inertia}}$$

Since we have $F_{\text{Newton}} = m a$ (Newton's Principles, law II)

it follows:

$$F_{\text{Inertia}} = - m a;$$

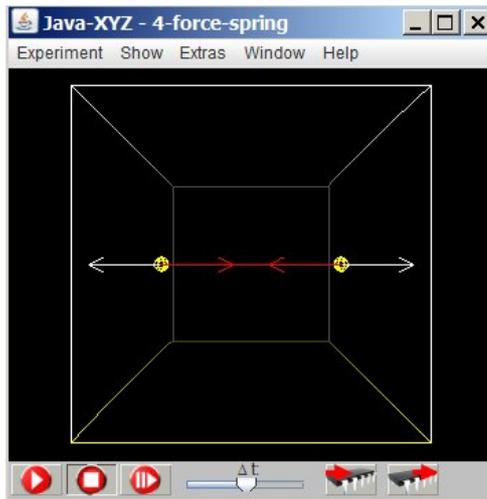
The equation $F_{\text{Newton}} = - F_{\text{Inertia}}$ is an expression for equilibrium and should not be identified with a static or stationary situation where two Newtonian forces are applied to the same object. In such a case, the sum of all applied Newtonian forces is zero and the object will remain in its state of rest, or uniform motion in a straight line (Newton's Principles, law I).

An equilibrium between a Newtonian and an acceleration-reaction-force, a force due to inertia, does only exist **during the process of acceleration**.

With the idea of inertia as an acceleration-reaction force, the **rule** to be learned is rather simple:

There exists always an equilibrium between two forces, either in a static or stationary situation between two Newtonian forces or during acceleration between a Newtonian force and a force of inertia.

4.6. Elastic Interaction and Force of Inertia



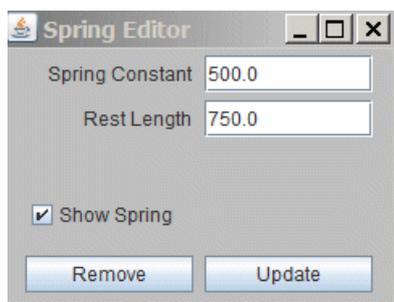
The simulation "4-force-spring" shows the movement of two objects connected by an elastic spring. Again it is possible to visualize the equilibrium between the Newtonian forces, exerted by the spring and the inertial forces as acceleration-reaction force.

A compressed spring is indicated as a red line, an extended spring as a blue line. A neutral spring is indicated as white line.

The simulation shows in accordance with reality that in every moment the inertial force F_{inert} is of equal strength as the Newtonian force F_N , applied by the spring. Both forces act in the same direction but with opposite orientation. $F_N = - F_{inert}$

Fig.4.5.: Simulation 4-force-spring

A compressed spring is indicated as a red an extended Spring as a blue and a neutral spring as w white line.



A spring can be set between two particles by 1. keeping the shift-key down and 2. by selecting one particle with the left mouse button down and release the button on top of the other particle. If this procedure is repeated over an existing spring, the "Spring Editor" window opens, where spring parameters can be set.

Fig.4.6.: Spring Editor window

Since the visualization of the force of inertia can be suppressed, it is always possible to explain Newton's law with inertia as a property of mass in the traditional way.

The simulation above can be used to find out, what will happen if the mass of both particle will be changed. The mass of a particle can be changed inside the "Particle Inspector" window. An explanation for the demonstrated effect is given in the next chapter.

4.7. Elastic Interaction - Hook's Law

A spring at rest without any applied force has a certain length l_0 .

When equal forces are applied at both ends the spring will either extend or shorten and change its length to l .

Due to this change of length the spring will react with a force of elastic origin $F_{elastic}$ which is (within limits) proportional to 2 terms:

- the difference between l and l_0 .
- a spring constant D which is specific for every spring and indicates how stiff or soft a spring is. The larger D the stiffer the spring.

These relations are represented in mathematical form as:

$$F_{elastic} = s D \quad (l - l_0 = s).$$

This law is called "Hook's law".

During the process of acceleration the length s and therefore also the force F are not constant but are changing in time.

The direction of the force is such that both particles will always be accelerated towards the position of rest, when the spring is without tension.

The larger the mass m of the particles the smaller is the acceleration due to the force of the spring (not depending on the mass of the particles).

As a result of an increased mass of the oscillating objects the oscillation will need more time for one cycle.

4.8. Force and Constant Velocity

If an object moves with constant velocity many people believe that it needs a constant driving force to keep the velocity constant. If for instance car drivers want to increase the speed of their car they have to increase the driving force of the motor by stepping on the gas pedal or have to lift their foot to slow down.

So the conclusion is: To keep up a movement with constant velocity, it needs a constant driving force.

This conclusion implies: If there is no driving force, the object will come to rest.

From a physics perspective these conclusions are problematic, because they are in contradiction to one of the basic laws in Mechanics - Newton's 2nd principle - stating that a constant force will cause a constant acceleration, a linearly increasing or decreasing velocity, depending on the orientation of the force

A series of videos have been developed which shows scenes from daily life, where objects are shown, moving with constant speed.

Vehicle on a country road http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/vehicle.htm

Gliding aeroplane http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/air_plane.htm

Ship in calm water http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/ship.htm

Motor boat on a river http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/boat.htm

Skydiver on an open parachute http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/skydiver.htm

A balloon floating in the wind http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/balloon.htm

Bubbles rising in a liquid http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/bubbles.htm

Slider on an air-cushion track http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/air_cushion.htm

The message of each of these videos is the same: If the sum of all applied forces cancel each other, so that the sum of all forces is zero, the object will move with constant speed.

5. Fall and Throw - Law of Gravitation

5.1. Weight - Weightlessness

In the middle of the 17. century Newton formulated the universal law of gravity, stating that all material objects are interacting in the form of mutual attraction (see glossary below).



Due to the gravitational interaction with the total mass of the earth, every material object has weight. But what about weightless conditions?

Does the child in the figure has weight or is it without weight? Only in this very moment or during the complete "throw"?

These questions and others related to movements under the influence of gravity will be answered on the following pages

Fig 5.1.: Is this child weightless?

5.2. Law of Gravity

Qualitative Description

The phenomenon of gravitation means that all material objects attract each other. Because of this fact all objects are attracted by the earth and have therefore weight. This seems to be a rather normal fact, however, until today the physicists have not found any explana-

tion for it. It must be accepted as a basic phenomenon which cannot be explained or reduced to a more basic principle.

The attracting force between two material objects 1 and 2 is proportional to a specific property of these objects, which we call here tentatively grav-property.

It has been shown experimentally that under all circumstances this grav-property is proportional to the former introduced term "mass". This fact is at first sight rather astonishing, because there is no obvious relation between the property of an object to resist acceleration - its inertial mass - and the property to attract another material object.

Einstein has developed an explanation in his General Theory of Relativity, which involves curved spaces and which can only be understood by specialists. There are other more speculative explanations, where inertia is reduced to a special gravitational effect, involving the galaxies of the

whole cosmos.

In any case it is now generally accepted that the strict proportionality of inertial mass and the above introduced grav-property indicates, that both have a common origin. Therefore both properties are nowadays called "mass". To distinguish between these two properties, they are often called "inertial mass" m_s and "gravitational mass" m_g .

Quantitative Description

A universal law can be stated: all material objects attract each other.

The attracting force between two material objects 1 and 2 is proportional to m_1 and proportional to m_2 . The attracting force also depends on the distance r between the attracting objects. Its magnitude is inversely proportional to the square of the distance between the centres of the two bodies.

Quantitatively, the gravitational law can be written as: $F_{1 \rightarrow 2} = -F_{2 \rightarrow 1} = \gamma \frac{m_1 \cdot m_2}{r^2}$

The constant γ is a universal constant of nature.

5.3. Objects in Free Fall

All objects - when starting from rest - fall to the ground with the same, constantly increasing velocity, independent of their weight (if air resistance is neglected). The video „Free Fall” (fig.5.2) shows an experiment where three different balls of quite different weight are dropped from a height of about 4m.

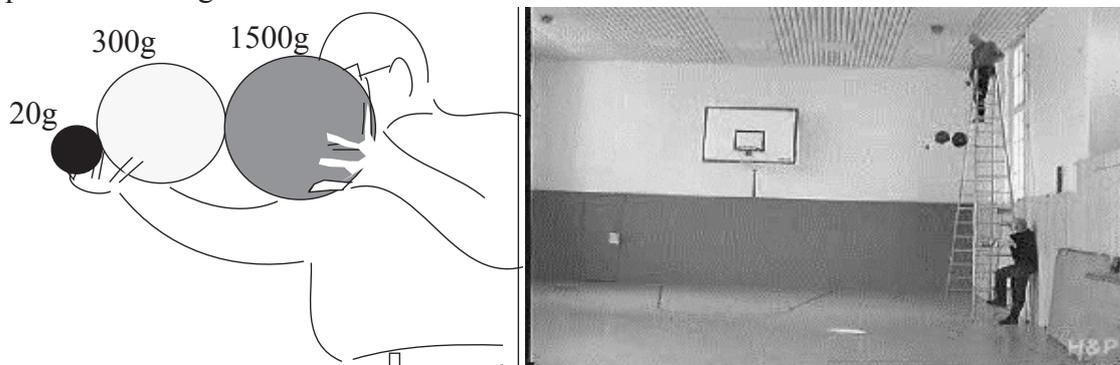
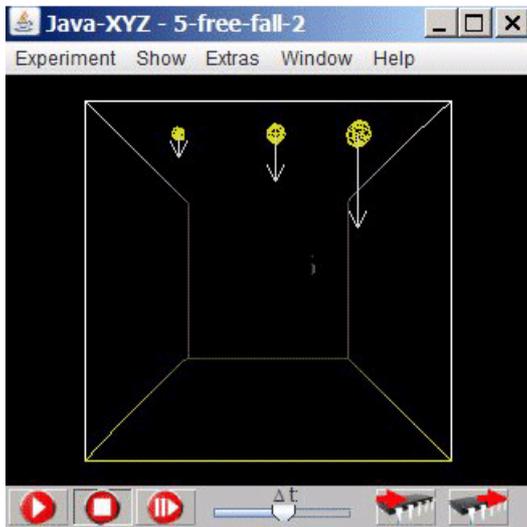


Fig 5.2.: Video about an experiment where 3 objects of different weight drop down (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/free_fall.htm)

For many people this may not be an astonishing fact because they take this as a law of nature. There are, however, many people who are convinced that smaller and lighter objects are dropping slower than larger and heavier ones. There is an astonishing point here.

A smaller object is pulled downwards by the earth with a smaller force than a larger object made from the same material. Why will nevertheless both objects fall along equal distances in equal length of time?



The answer is found in the fact that the object with a smaller gravitational mass has also a smaller inertial mass. Therefore its resistance against acceleration is less. And since gravitational mass and inertial mass are strictly proportional, these two effects - less attracting force and less inertia - cancel for every object and every amount of mass.

The prepared simulation "5-free-fall" demonstrates that the program JavaXYZ is in agreement with reality. This is of course not astonishing. Astonishing is the fact that inertial mass and gravitational mass are proportional

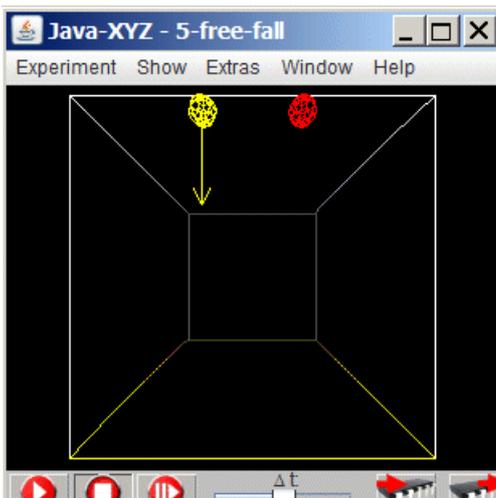
Fig 5.3.: Simulation "5-free-fall"

5.4. Drop Distance and Drop Time

All objects experience the same acceleration when dropping down to earth. This implies that a general relation exists to determine the drop distance from a given drop time which is valid for all objects, independent of their weight.

For the simple case that an objects is released from rest this relation reads:

$$s = 1/2 g t^2; s = \text{drop distance}; g = \text{acceleration due to gravity}; t = \text{drop time}$$



The simulation "5-free-fall-2" explains, how this relation can be derived.

In this simulation an electric field is applied in negative z-direction (downwards) but no gravitational field exists. Since the right particle is charged, it moves under a constant force and therefore with a constant acceleration a_c . The left neutral particle moves due to its starting velocity with constant speed v_c .

Question: What constant velocity v_c is necessary for the body on the left so that both reach the bottom of the cube at the same moment in time?

Fig 5.4.: Comparison of 2 objects moving with constant speed and constant acceleration Simulation "5-free-fall-2"

The starting velocity of the left particle can be changed in the "Particle Inspector" window (fig 3.1).

Result:

The velocity of the accelerated body increases from zero to a maximum value v_{\max} when it hits the bottom of the cube.

To cover the same distance s_{\max} during the same time the body on the left must move with a constant velocity $v = v_{\max}/2$. With this starting velocity it is too fast at the beginning and too slow at the end. Since the velocity of the accelerated body does increase linearly from zero up to v_{\max} , this "too slow" and "too fast" will balance and both objects will reach the bottom at the same moment in time

The body on the left, moving with constant velocity covers during the time period t_{\max} the

$$\text{distance: } s_{\max} = \frac{v_{\max}}{2} \cdot t_{\max}.$$

For a constant acceleration we get for the velocity as function of time $v_{\max} = g \cdot t_{\max}$.

$$\text{For the distance } s_{\max} \text{ we get: } s_{\max} = \frac{g}{2} \cdot t_{\max}^2$$

This relation is valid for arbitrary distances and time spans as long as the object starts from rest. Therefore the index can be removed and we get as relation between distance to fall

$$\text{and time to fall: } s = \frac{1}{2}g \cdot t^2$$

The simulation "5-free fall" with the parameters: $s_{\max} = 2000$ units; $g = 10$ units shows an agreement between the mathematical relation given above (with $v = v_{\max}/2$)

5.5. Free of Weight and Free of Force - An Important Difference

When looking at astronauts as they are floating in their space craft while circulating around the earth, it is obvious that they are weightless. But are they also free of force? Certainly not, because these space crafts move within the gravitational field of the earth. For usual orbits the strength of this field is nearly the same as here on earth.

The same is valid for an object in free fall. It certainly is weightless, otherwise scientists would not have build a so-called drop tower at University Bremen, Germany, where low-cost experiments under conditions of weightlessness are carried out.

Look at the video and try to find the answer to the question posed about a throw upwards.

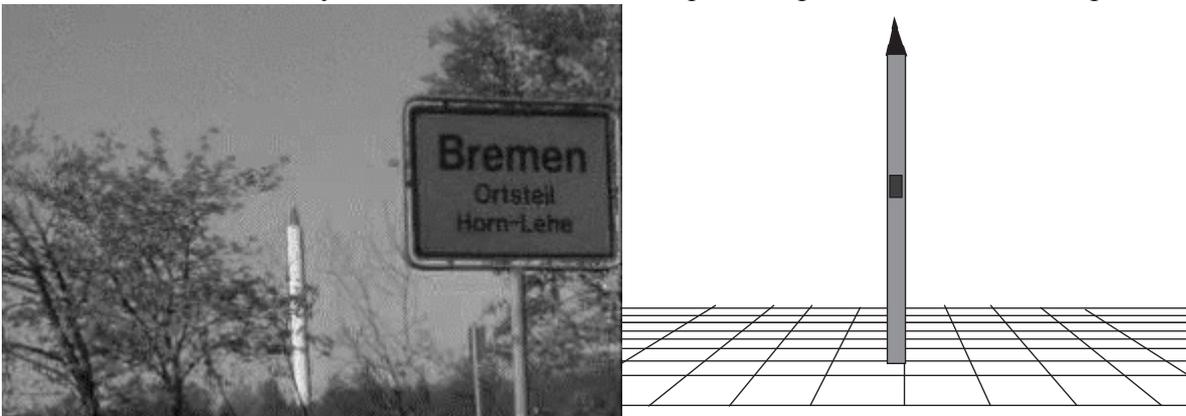


Fig 5.5.: Video about the drop-tower at Bremen

http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/fallturm_en.htm

Answer

An object has weight when it is in equilibrium between two forces, gravity pulling downward and a counter force, applied by the ground, pushing upward.

A scale indicates as weight either this force downward or this counter force which is necessary to keep up the equilibrium.

Due to these two forces, oriented downward and upward, an object will be more or less deformed depending on its elasticity. This elastic deformation disappears when there is no counter force from the ground but when only the gravitational force oriented downward is applied.

Gravity applies to every single atom or molecule of any object in the same manner and therefore does not create any tension or elastic deformation. It is this non-deformed state which is interesting for scientist. Therefore such experiments under weightless conditions are carried out, either in a drop tower or in space crafts like the ISS.

Why objects in space crafts are weightless will be explained in later chapters.

5.6. Vertical Throw**Starting Velocity and Time to Rise**

The prepared simulation "5-vertical-throw" allows to find out which starting velocity upward is necessary for the ball to move against gravity and to reach the top of the cube without bouncing against it. Simulated is a constant gravitational field downwards.

Such a gravitational field can be activated in the Global Parameter window.

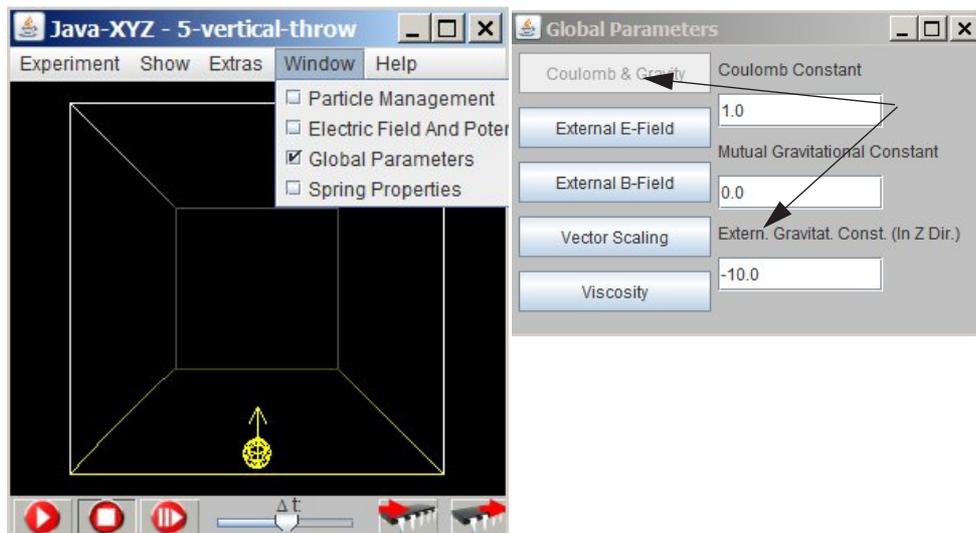


Fig 5.6.: Simulation "5-vertical-throw"

How to set and change an external gravitational field in z-direction

A vertical throw and a vertical fall are symmetric movements with constant acceleration. Therefore the same equations have to be applied.

$$s = \frac{1}{2} g t^2; v = g t + v_0$$

In the simulation the value for the acceleration is $g = -10$ units.

Is there an agreement between the results of the simulation and these equations?

5.7. Horizontal Throw

The Experiment and a Question

The video (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/ho_throw.htm) shows an experiment, where a ball is pushed horizontally from a height of about 4 m and hits a basket full of water at a distance of about 6 m.

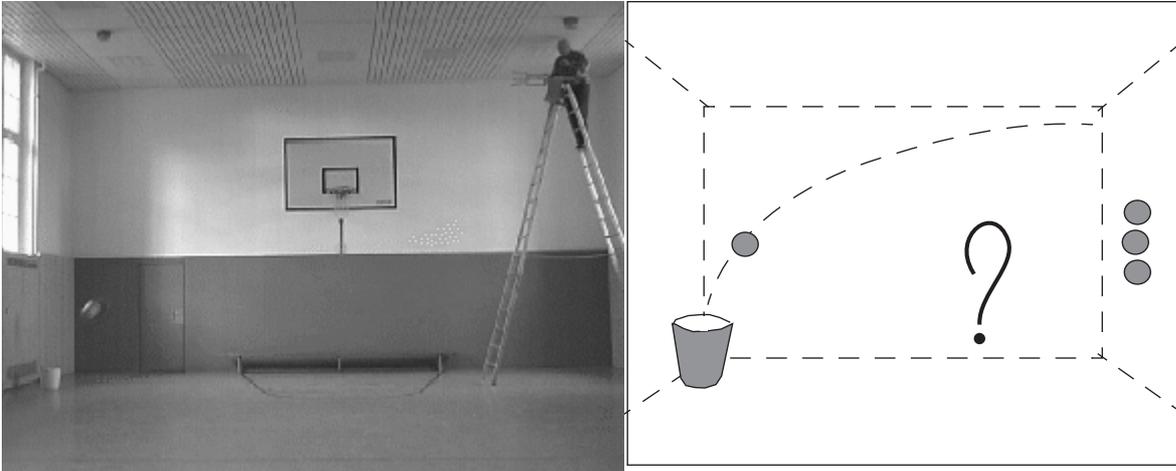


Fig 5.7.: Video about a horizontal throw

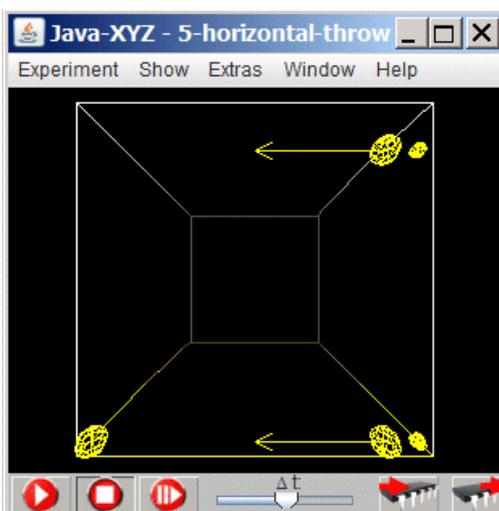
Question:

If a second ball would start to fall vertically at the same moment in time when the ball is pushed horizontally, would it reach the floor before or later or at the same moment in time?

A first answer can be found by using the prepared simulation "5-horizontal-throw" (next chapter).

Simulation - Principle of Superposition

It would not be a simple task to set up a real experiment where the two balls are released at the same moment in time with variable or no horizontal velocity components. To develop a corresponding simulation is rather simple and is sufficient, because of a basic principle - the so-called superposition principle. This principle is based on the fact that a force has absolutely no influence in a direction perpendicular to its own direction.



In our case this means that gravity cannot change the horizontal velocity component in any way. This component of the velocity remains constant - it will at least not be influenced by gravity - while the acceleration downward will be the same as for all free falling objects. The two balls will therefore - as demonstrated by the simulation - always hit the ground at the same moment in time. This will happen in any real experiment with an accurate experimental setup.

You may find out by trial and error, that a specific velocity v_0 is the right one to hit the ball on the left.

Fig 5.8.: Simulation 5-horizontal-throw
(with a simulated gravitational field downwards)

The simulation indicates in accordance with reality that the movement of the horizontal throw is an independent superposition of a horizontal component with constant velocity and a vertical component with constant acceleration.

An alternative to trial and error is an approach based on theory as shown below.

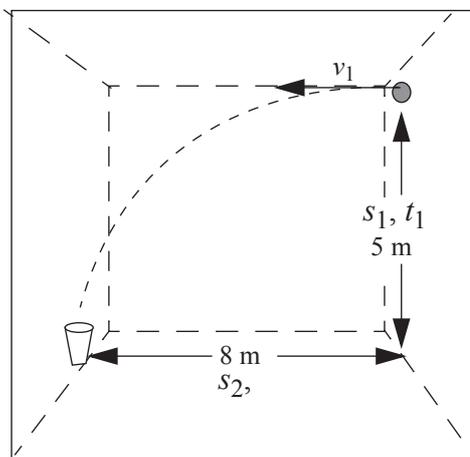
Numerical Solution

In the video the question was posed; What starting velocity is needed for the ball to hit the basket (distance 8m) from a height of 5m?

To answer this question, the following relations are needed and have to be understood:

- For a movement with constant acceleration: $s = \frac{1}{2} g t^2$; $v = a t$; (if started at rest).
(s = distance travelled; v = velocity; g = acceleration; t = time span)
- For a movement with constant velocity: $s = v t$.

From the principle of superposition it follows that the movement shown in the video and by the simulation can be separated in two parts, a free fall with constant acceleration $a = g$ and a horizontal movement with constant velocity v .



For the free fall in perpendicular direction we have:

$$s_1 = \frac{g}{2} t_1^2 \quad t_1 = \sqrt{\frac{2s_1}{g}}$$

$$s_1 = 4m; g = 10m/s^2; t = \sqrt{0,8}s$$

For the horizontal movement we have:

$$v_2 = \frac{s_2}{t_1} = \frac{6}{\sqrt{0,8}} \frac{m}{s}$$

The values for the simulation are:

$s_1 = 1800$ units; $s_2 = 1800$ units; $g = -100$ units.

Is there an agreement between theory and simulation?

5.8. Inclined Throw

The Experiment

The inclined rod, seen in the video, is hold under tension by two strong springs. When released, the top of the rod hits the ball and after quite a few tryouts with different starting positions and different inclinations the ball reached the basket and made a point.

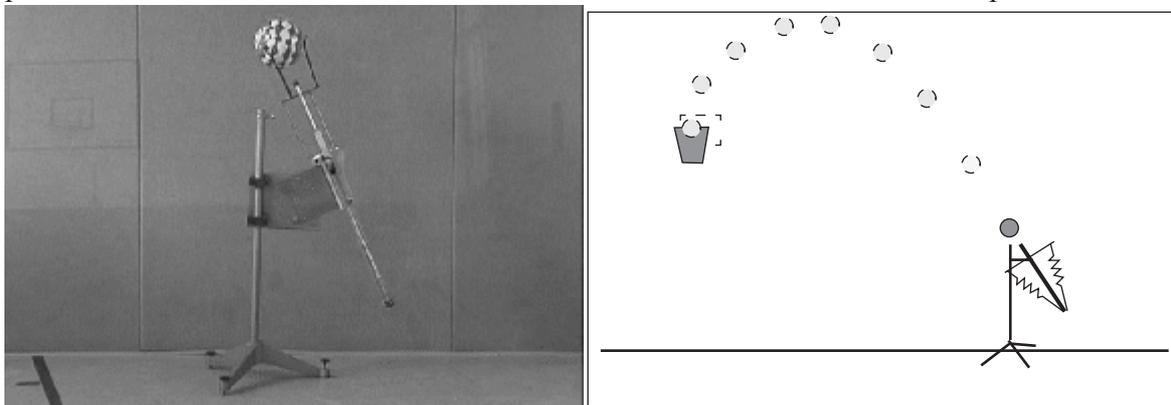


Fig 5.9.: Video at http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/incl_throw.htm

The next simulation repeats this experiment, followed by a numerical solution.

The Simulation

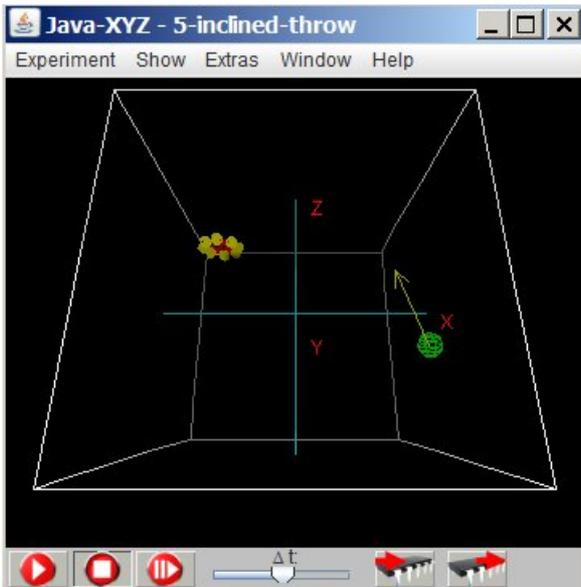


Fig 5.10.: Simulation 5-inclined-throw

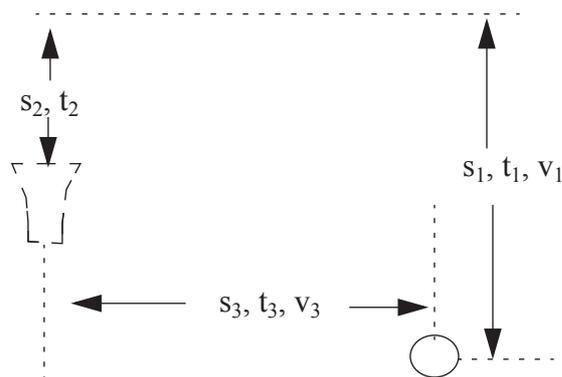
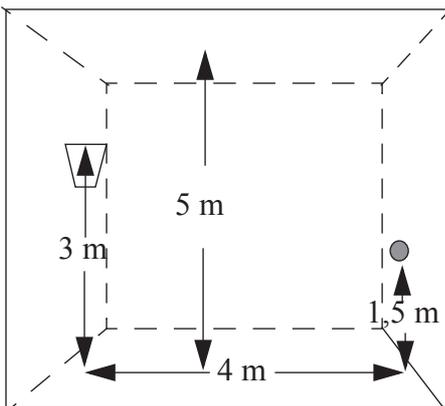
Numerical solution:

To solve numerically the problem posed in the video, the following relations are needed and have to be understood:

- For a movement with constant acceleration a_c : $s = 1/2 g t^2$; $v = g t$;
(s = distance travelled; v = velocity; g = acceleration due to gravity; t = time span)
- For a movement with constant velocity v_c : $s = v t$.

From the principle of superposition it follows that the movement shown in the video and by the simulation can be separated in two parts, a vertical throw with constant acceleration and a horizontal movement with constant velocity.

For the two parts of the vertical throw with velocity upward and downward we have:



$$s = \frac{g}{2} \cdot t^2 \quad t = \sqrt{\frac{2s}{g}} \quad v = g \cdot t$$

$$s_1 = 3,5 \text{ m} \quad s_2 = 2 \text{ m} \quad g = 10 \frac{\text{m}}{\text{s}^2}$$

$$t_1 = \sqrt{0,7} = 0,83 \text{ s} \quad t_2 = \sqrt{0,4} = 0,63 \text{ s} \quad v_1 = g \cdot t_1 = 8,3 \frac{\text{m}}{\text{s}}$$

By changing the horizontal and vertical components of the starting velocity, you may try to hit the "basket" but as much as possible from above, without hitting the "ceiling".

An alternative to trial and error is an approach based on theory.

For the horizontal movement we have:

$$s_3 = 4\text{m}; \quad t_3 = t_1+t_2 = 1,46\text{s}; \quad v_3 = s_3/t_3 = 4/1,46 = 2,74 \text{ m/s}$$

The values for the simulation are:

$$s_1 = 1250 \text{ units}; \quad s_2 = 450 \text{ units}; \quad s_3 = 1600 \text{ units}; \quad g = -100 \text{ units}.$$

Is there an agreement between theory and simulation?

5.9. A Long Stick and a Ball Dropping Down

It has been shown that all objects experience the same acceleration when dropping down to the earth. If confronted with an experimental setup as seen in the following video, you may at first sight assume that the stick and the ball, when released simultaneously, will hit the ground at the same moment in time.

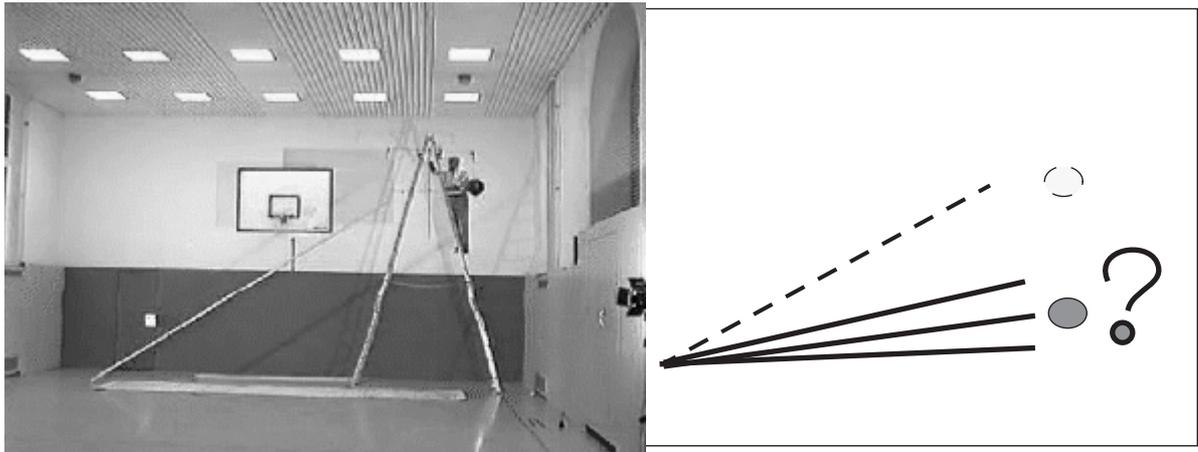


Fig 5.11.: Video about a dropping stick and a ball
http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/stick_f.htm

Are you astonished when you observe the outcome of this experiment? (see the video at http://www.astrophysik.uni-kiel.de/~hhaertel/CGA/Flv/stick_e.htm)

The puzzling question is: What is the extra force (in addition to gravity), which causes the upper end of the stick to accelerate more than the free falling ball?

Stick and Ball (Explanation)

A first answer is a rather simple one. In this experiment two different cases are compared. The ball is under the influence of gravity only and therefore it is in free fall. The inclined stick, however, is not only under the influence of gravity but is also in interaction with the floor. It is therefore not in free fall and so we should not expect a simple result.

By experimenting with the prepared experiment „Stick and Ball“ you can learn more about the processes related with this experiment.

Stick and Ball (Simulation)

Similar to the video the simulation "5-falling-stick-ball" allows to compare the free fall of a ball and the movement of an inclined stick. The stick is modelled by single particles, connected by elastic springs.

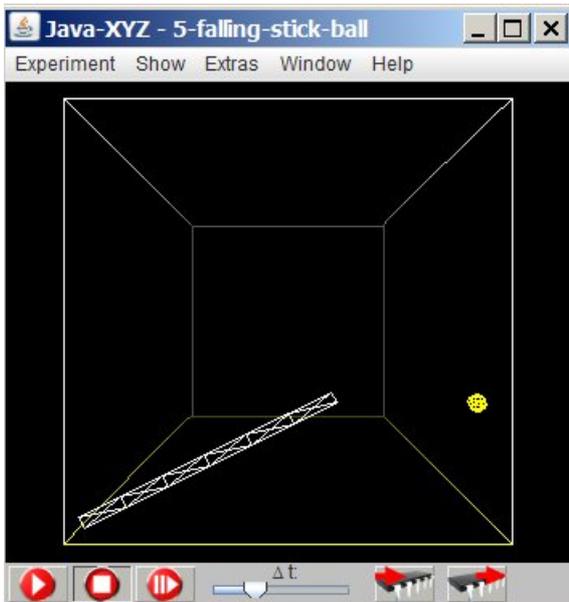


Fig 5.12.: A falling stick and a falling ball
(simulation "5-falling-stick-ball")

5.10. A Soft Spring Dropping Down

Assume that your neighbour upstairs is bending out of his window and holds the upper end of a rather soft spring, a so-called Slinky, in his hand. The Slinky extends under gravity and ends up in front of your window, where you only see the lower end of the spring.

The question is: What will happen to this lower end, when your neighbour releases the spring? Will it move upwards a little, will it stay for a short while before starting to drop or will it immediately start dropping?

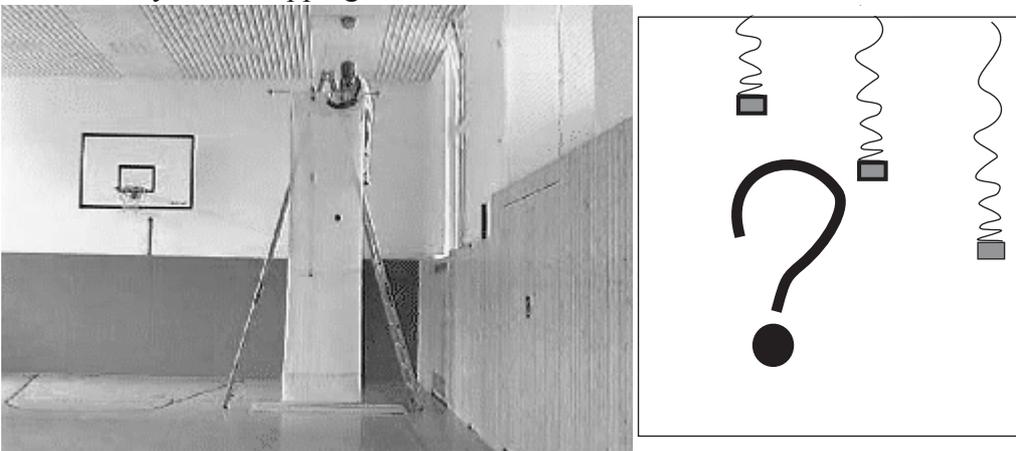


Fig 5.13.: Video about a dropping elastic spring (a Slinky)
(http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/spring_f.htm)

Look at the video indicated in fig.5.13 and make up your mind about the question posed.

If you look carefully at the next video (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/spring_e.htm), which shows the experiment, you can detect that the lower end of the Slinky does not start immediately but only when the spring has nearly collapsed.

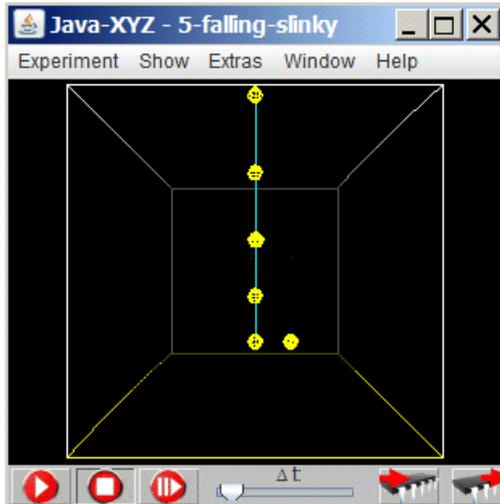
This process can be studied in slow motion with the simulation "5-falling-slinky"

While the left corner of the stick is fixed, the other parts are forced to move on more or less circular paths around this fixed point.

Due to the two forces, gravity and the reaction from the floor, the stick is bending and increasing elastic forces lead to increased acceleration.

If the elasticity is reduced by applying more springs, (simulation "5-falling-stiff-stick-ball") the bending is nearly invisible.

The stiffer the stick the better the agreement between the experiment and what is shown in the simulation.

Simulation "5-falling-slinky"

In this simulation a Slinky is modelled as a sequence of particles of equal mass connected by short elastic springs of equal spring constant.

The particle on top is fixed.

Since gravity is active the spring is expanded under its own weight. The single particle next to the lower end of the Slinky does not fall because it is fixed.

If you start the simulation and release the fixed particles before, you can observe when the lower particle starts falling.

Fig 5.14.: Simulation of a falling slinky

To fix or to release a particle can be controlled inside the "Particle Inspector" window

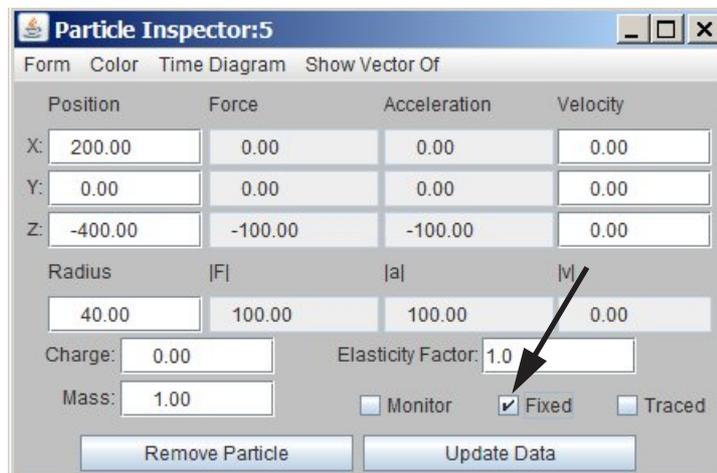


Fig 5.15.: How to fix or release a particle

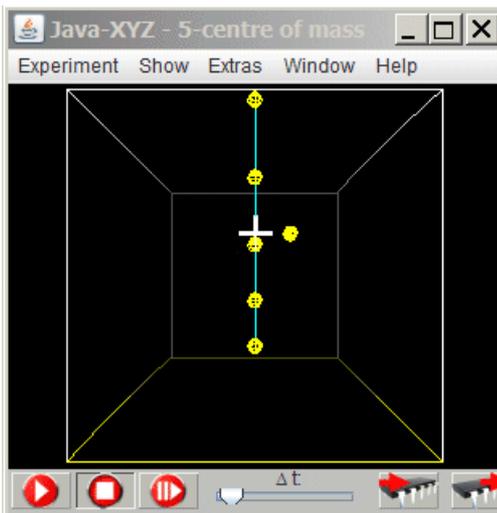
Result:

The lower particle cannot start to fall immediately because the spring has not yet collapsed and therefore compensates gravity.

5.11. Concept of Centre of Mass

With the following simulation the concept of "Centre of Mass" (CoM) is introduced. This concept will not be derived from more basic principles but is just demonstrated to show its unique property and to motivate for more extended study.

In the simulation "5-centre of mass" the Slinky is modelled as before as a sequence of particles of equal mass connected by short elastic springs of equal spring constant.



The particle on top is fixed. Since gravity is active all the springs are expanded under the weight of the particles.

The white cross in this simulation indicates the so-called centre of mass of the Slinky.

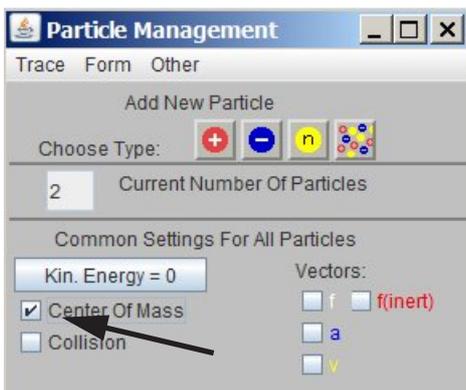
The single particle next to the centre of mass of the Slinky does not fall because it is fixed.

If you start the simulation and release before the fixed particles (fig. 5.11), you can observe that the centre of mass moves exactly at the same pace as the free falling body.

Fig 5.16.: Simulation "5-centre-of-mass"

The general rule is:

The centre of mass of an extended material object behaves under the influence of an external force as if all the mass of the extended object is concentrated at this point. Internal forces have no influence on its movement.



The centre of mass of all particles placed inside the cube can be visualized inside the "Particle Management" window

Fig 5.17.: How to set the centre of mass

6. Circular Motion

6.1. Centripetal Force

Newton's 1st law reads as follows:

Every body remains in its state of rest, or uniform motion in a straight line, unless it is compelled to change that state by an applied force.

Important here is the word: "straight line". If a body does not move on a straight but on a curved line, its velocity is changing its direction. This is not possible without an applied Newtonian force, even if the object moves with constant speed.

For a homogeneous movement on a circular path, where the curvature is constant, the applied force F - named centripetal force - is directed in every moment towards the centre of the circle.

For the force F , needed for an object to move with constant speed on a circular path the following relation can be derived from Newton's 2nd law as:

- $F = m a = m v^2/R$ ¹
 m = inertial mass of the circulating object
 R = radius of the circular path
 $|v|$ = constant amount of the velocity on the circular path.

The derivation of this equation can be found in any text book.

As already mentioned this force is called „centripetal force“. In every moment it is oriented towards the centre of the circular path.

What about the well-known „centrifugal force“ oriented towards the outside?

The following video poses an interesting question related to these two kinds of forces.

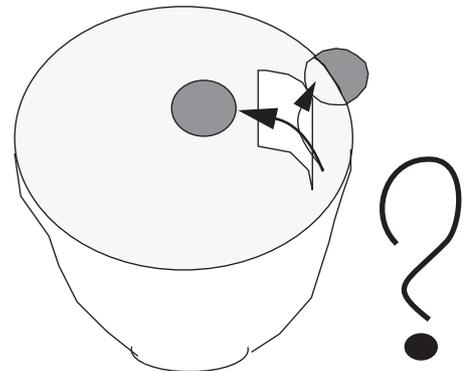


Fig 6.1.: Video "ball in a rotating beaker"

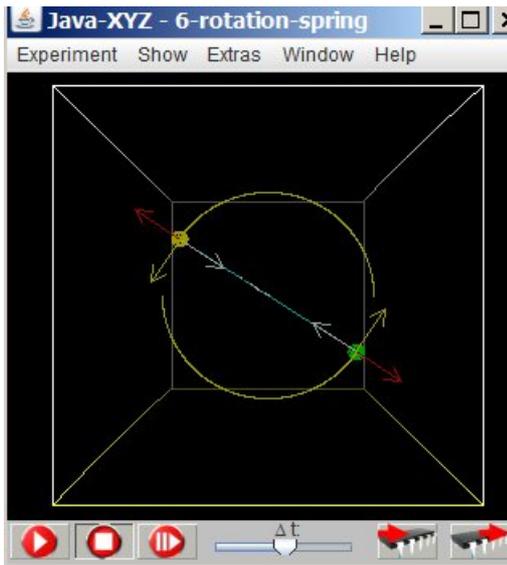
(http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/ball_beaker.htm)

Will the ball stay inside or go outside, due to the so-called „centrifugal force“?

More related information is found on the next pages.

1. The derivation of this equation is found in every standard textbook.

6.2. Circulating Objects, Connected by an Elastic Spring



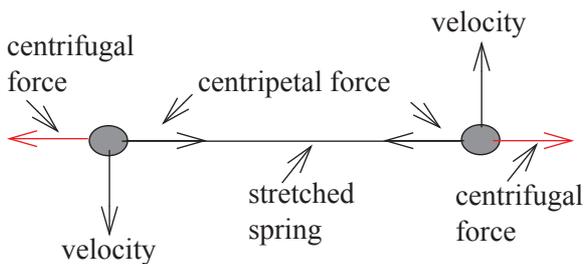
The simulation "6-rotation-spring" shows the movement of 2 particles, connected by a spring and circulating around a common centre.

For moving on a circle with radius R and constant speed a centripetal force is needed, oriented towards the centre of the circular path. This force is provided by the stretched spring.

If the mass and the amount of the velocity (its speed) is changed, the orbit of the two particles is no long circular. Mass and velocity can be changed in the "Particle Inspector" window (fig. 3.1). The same window allows to initiate tracing for a specific particle or to stop tracing.

Fig 6.2.: Simulation "6-rotation-spring"
2 objects connected by a spring

Force of Inertia - Centrifugal Force



As always for acceleration processes, a force of inertia as acceleration-reaction force can be visualized (fig. 4.3 for all particles to be set; fig.4.4 for a selected particle already set).

For circulating movements this force is known as "centrifugal force".

Accelerated movement on a straight line can be interpreted as an equilibrium between a Newtonian and an acceleration-reaction force. The same is possible for movements on a curved path, in this case a circular path. In every moment the centrifugal force as an acceleration-reaction force F_{a-r} is of equal strength as the centripetal force F_N , a Newtonian force. Both forces act in the same direction but with opposite orientation:

$$F_N = - F_{a-r} \text{ or } F_{\text{centripetal}} = - F_{\text{centrifugal}}$$

Computation

To achieve a circular path of both particles this force has to correspond to the centripetal force $F = m v^2/R$ needed for each particle to circulate with a velocity $|v|$ around a centre with radius R .

The force which the stretched spring applies on both particles can be determined by Hook's law:

$$F_{\text{spring}} = s D$$

(l_0 = rest length; l = actual length under tension; $s = l - l_0$; D = spring constant)

$$F_{\text{spring}} = s \cdot D \quad F_{\text{centripetal}} = m \cdot \frac{v^2}{R} \quad R = \frac{l_0 + s}{2}$$

$$s \cdot D = m \cdot \frac{v^2}{R} = 2m \cdot \frac{v^2}{l_0 + s} \quad v^2 = \frac{s \cdot D}{2m} (l_0 + s)$$

The settings in the simulation are (in corresponding units): $l_0=1500$; $s=100$; $m=2$; $D=1$;

You may calculate the value for $|v|$ for a circular orbit and compare it with the simulation.

For $m \neq 2$ or for different spring parameters the relation given above for a circular motion is no longer valid. The orbit of the particles is no longer strictly circular.

Video „Ball in a rotating Beaker“

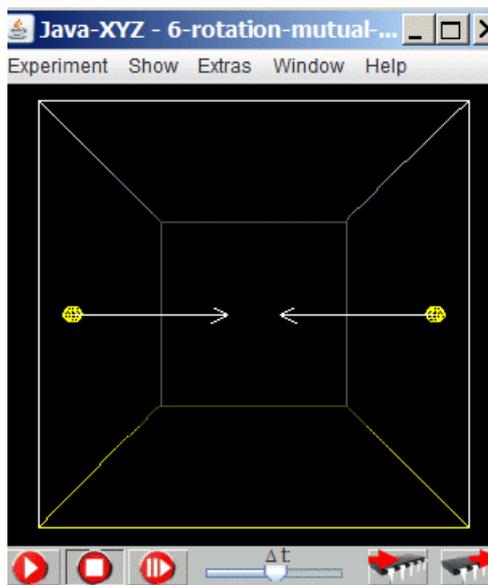
In the experiment shown in the video above, the centripetal force on the ball is applied by the wall of the beaker and oriented towards the centre.

When the ball reaches the hole in the wall, this centripetal force disappears. The movement is no longer accelerated and so the centrifugal force - an acceleration-reaction force - also disappears. The ball will continue to move on a straight line.

If the width of the hole is not too large the ball will hit the opposite border of the hole from the inside and will stay inside of the beaker.

6.3. Objects Moving under Mutual Gravitational Attraction

The simulation "6-rotation mutual-gravity" allows to study the situation where 2 particles are rotating on a circular path due to mutual gravitational attraction.



In the starting position the particles are at rest and will accelerate towards each other.

If a velocity with opposite orientation is applied to each particle in the direction perpendicular to the connecting line (in this case the z-direction), the particles will start moving on orbits around a common centre.

For a specific velocity, depending on the applied forces and the mass of the particles, a circular orbit can be reached.

Fig 6.3.: Simulation "6-rotation-mutual-gravity"

The corresponding velocity v_{circle} for a circular orbit can be found by trial and error. The correct value (in suitable units) for the given situation is for the right particle $v_z=100$, for the left particle $v_z=-100$.

The following computation shows how the velocity v_{circle} , necessary for an exact circular orbit, can be determined.

Computation of v_{circle} for a Uniform Circular Motion

Based on the general law of gravity and Newton's 2nd law the following relation can be derived for the path velocity of 2 gravitating objects circulating around a common centre.

$$F_{\text{gravitation}} = \gamma \frac{m_1 \cdot m_2}{r^2} \quad F_{\text{centripetal}} = m_1 \cdot \frac{v^2}{R} \quad R = \frac{r}{2}$$

$$\gamma \frac{m_1 \cdot m_2}{r^2} = m_1 \cdot \frac{v^2}{R} = 2m_1 \cdot \frac{v^2}{r} \quad |v| = \sqrt{\frac{\gamma \cdot m_2}{2r}}$$

For the given simulation we have the following values (in corresponding units):

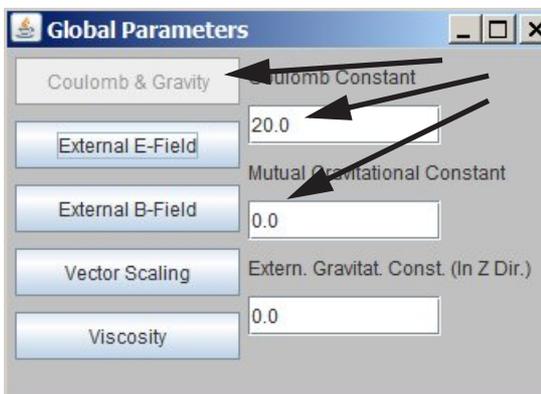
$$\gamma = 10^4; m_1 = m_2 = 10^3; R = 5 \cdot 10^2; v = 100$$

The simulation "6-rotation mutual-gravity" demonstrates an agreement between computation and simulation?

6.4. Movement of an Object in the Coulomb Field of a Central Body

Material objects in space, like earth, moon, the planets, the stars in our galaxy a.o. are always rotating in one way or the other. The following pages show why such rotation is necessary for system of material objects to be stable in space.

The simulation "6-orbit-coulomb" shows a small ball circulating around an attracting central body (mass ratio 1:10⁵). Due to this large ratio, the problem is very much simplified, because the movement of the large object is rather small and can be neglected. Both objects are charged with opposite polarity and are therefore attracting each other. The advantage of such a simulation becomes obvious if the effect of the inertial mass on the resulting orbit shall be studied



The charge of the particles can be set in the "Particle Inspector" window. To simulate such an attraction a Coulomb constant has to be set to a certain value. This can be done in the "Global Parameter" window. In the same window the gravitational constant can be set.

Fig 6.4.: How to set or change the Coulomb and gravitational constant

In reality, when objects rotate around each other (like moon and earth) due to mutual gravitational interaction, it is not possible to change the attractive force independent of the mass of the objects. In the simulation above, however, the attracting force between the particle is simulated as a Coulomb force. Therefore such independent variation of attractive force and mass is possible. This allows to pose some interesting questions, which could lead to a deeper understanding of rotating objects under mutual attraction.

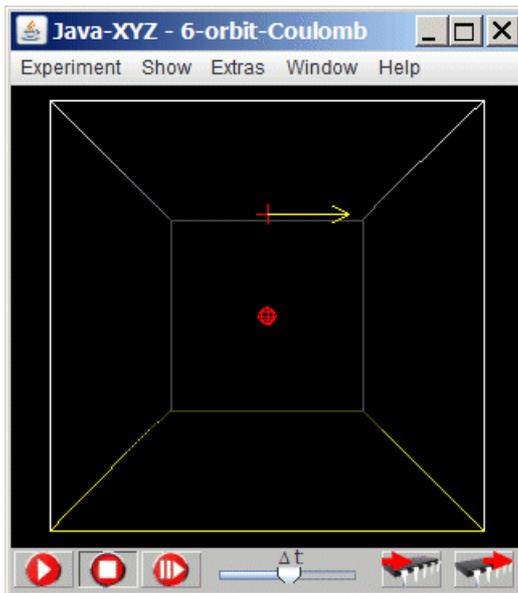


Fig 6.5.: Simulation "6-orbit-Coulomb"

How will the orbit of the circulating body (in the simulation "6-orbit-Coulomb") change if either the centripetal force (charge q of the circulating object) - directed towards the centre - or its mass m or both will be changed, but each time independent from each other?

When will the orbit become narrower, when wider?

When will the orbit stay the same?

Interesting settings are (in suitable units):

$q=10$; $m=13$

$q=13$; $m=10$

$q=13$; $m=13$ or $q=50$; $m=50$

6.5. Movement of an Object in the Gravitational Field of a Central Body

The simulation "6-orbit-gravity" shows a small ball circulating around a central body, attracted by gravity (mass ratio 1:106).

At start the velocity is set in such a way that the small body orbits on a circle.

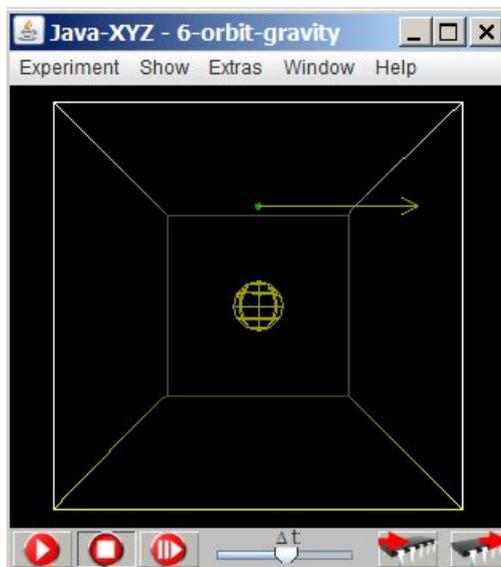


Fig 6.6.: Simulation "6-orbit-gravity"

We can also say: The circulating body is attracted by the central body and should drop down towards the centre. However, due to its tangential velocity it is always missing it. Therefore the small object is permanently free falling.

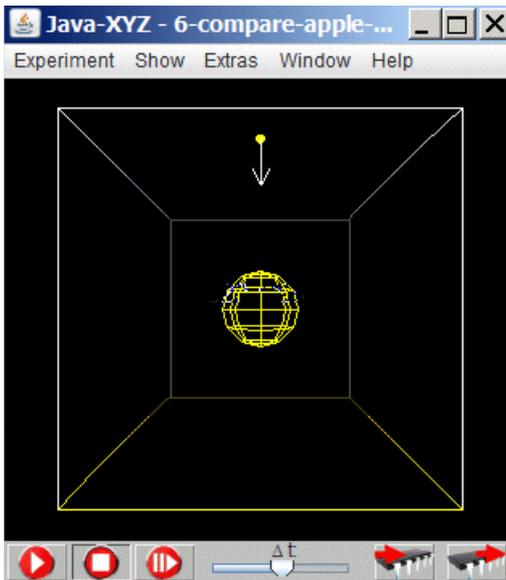
How will the orbit of the circulating body change if its mass is changed?

Will it stay the same, will it become wider or narrower. In case it stays the same, why?

6.6. A Dropping Apple and the Circulating Moon - a Comparison

When an apple drops down from a tree it moves perpendicular to the surface of the earth. Taking the earth as a system of reference at rest, the moon is orbiting around the earth on a nearly perfect circular orbit. These two movements rely on the same basic law. In the following will be shown, where the difference is.

The simulation "6-compare-apple-moon" shows first a satellite above a central object, for instance the earth. Without a tangential velocity it drops down on a straight line towards the central body.



If the tangential velocity v_x is increased stepwise, the satellite will move on kind of parabolic paths until it finally will start circulating on elliptic orbits with a circular orbit as special case

Interesting settings are (in suitable units)

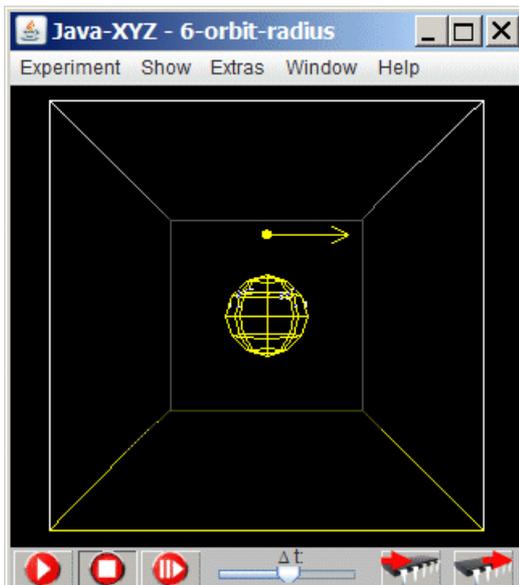
$$v_x=50; v_x=75; v_x=100; v_x=150$$

This simulation should be helpful to find the answer for the following question:

- What kind of common aspect exists between the movement of an apple, dropping down from a tree and the movement of the moon, circulating the earth?

Fig 6.7.: Simulation "6-compare-apple-moon"

6.7. Circulating Satellites on Different Orbits



The simulation "6-orbit-radius" shows a satellite, with a starting velocity of 200 units, and a distance from the centre of the central objects of 400 units.

If this distance is enlarged, a different velocity is needed to reach a circular orbit.

Instead of trying to find this suitable velocity by try and error, it is more efficient to do so by computation

Fig 6.8.: Simulation "6-orbit-radius"

Computation

As always in mechanics F_c and a_R follow Newton's basic law: $F_c = m a_{\text{radial}}$.

For the relation between radial acceleration a_{radial} , velocity $|v|$ and radius of the orbit R we have:

$$a_{\text{radial}} = v^2 / R.$$

For the centripetal force F_c it follows that:

$$F_c = \frac{m(\text{tr})_{\text{Satellite}} \cdot v^2}{R}$$

This necessary centripetal force is provided by the gravitational attraction between the satellite and the central body. The gravitational force is given by the universal law of gravity:

$$F_{1/2} = -F_{2/1} = \gamma \cdot \frac{m(\text{grav})_1 \cdot m(\text{grav})_2}{R_{1/2}^2}$$

If applied to a satellite, surrounding a central object we get:

$$F_{\text{earth-satellite}} = \gamma \cdot \frac{m(\text{grav})_{\text{earth}} \cdot m(\text{grav})_{\text{satellite}}}{R_{(\text{earthcenter/satellite})}^2}$$

In doing so we refer to the fact that a spherical body (such as the earth) behaves to the outside as if all its mass is concentrated at its centre. The distance therefore must be taken from the centre of the central body.

Since the centripetal force, necessary for a circular orbit is realized by this gravitational force we get:

$$F_{(\text{earth/satellite})} = F_c$$

$$\gamma \cdot \frac{m(\text{grav})_{\text{earth}} \cdot m(\text{grav})_{\text{satellite}}}{R_{\text{earthcentre/satellite}}^2} = \frac{m(\text{inert})_{\text{satellite}} \cdot v^2}{R_{\text{orbit}}}$$

From the fact that inertial and gravitational mass are proportional and measured in equal units it follows:

$$\gamma \cdot \frac{m(\text{grav})_{\text{earth}}}{R_{\text{earthcentre/satellite}}^2} = \frac{v^2}{R_{\text{orbit}}}$$

With $R_{\text{centre of central object/satellite}} = R_{\text{orbit}}$ and solving for v we get:

$$|v| = \sqrt{\gamma \cdot m_{\text{earth}}} \cdot \sqrt{\frac{1}{R_{\text{earthcentre/satellite}}}}$$

If velocity v_1 and radius R_1 for a specific orbit are known and if the velocity v_2 for a different radius R_2 needs to be calculated, we have: $v_1/v_2 = \sqrt{R_2/R_1}$.

Example: From $R_2/R_1 = 2$ follows: $v_2 = v_1/\sqrt{2}$.

The simulation "6-orbit-radius" allows to check this computation by changing the distance between satellite and central object as well as the corresponding starting velocity of the satellite. Both parameters can be changed inside the "Particle Inspector" window.

7. Collision and Conservation of Momentum

7.1. Introduction

The following chapter is centred around collision processes where the term “momentum” plays an important role. The term momentum can be defined for any moving object. It is indicated as p and is defined as the product of mass m and velocity v : $p = m \cdot v$

Just as with energy there exists a law of conservation of momentum, which has universal validity.

The term “momentum”, which is of high importance within classical as well as modern physics, will be introduced and explained by treating collisions processes of particles.

The video (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/collision.flv) shows such a collision process between 2 solid metallic balls and the smooth transition to a corresponding simulation.

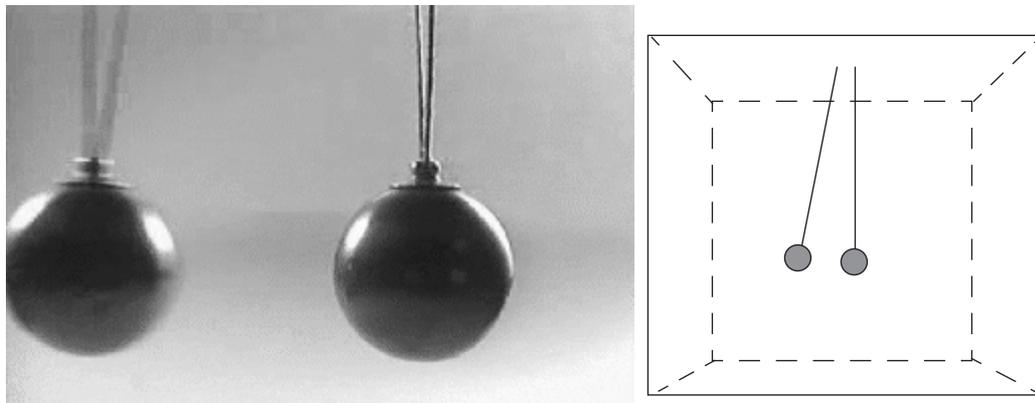
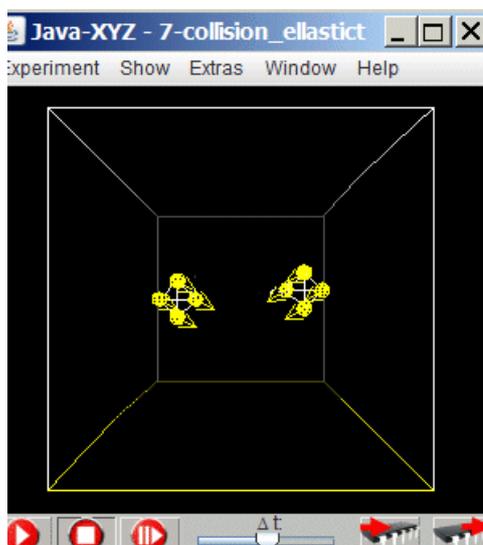


Fig 7.1.: Video about collision

The following simulations allow to study such collision processes in detail. Real experiments with sliders on an air cushion track or solid balls like in the video are the base of such simulations and should, if at all possible, precede or accompany any further study.

7.2. Collision with Realistic Objects



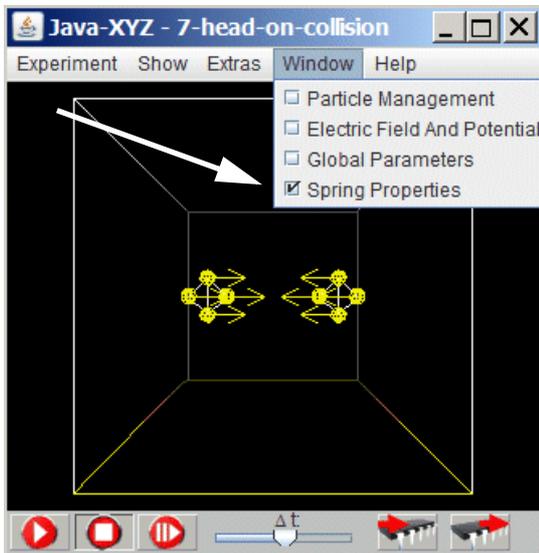
The simulation "7-collision-elastic-objects" shows a collision between two bodies, modelled as extended objects of 4 particles, interconnected by elastic springs.

As a normal result of every collision process the colliding bodies change their velocity. In addition internal vibrations are excited and the bodies may start to rotate.

Fig 7.2.: Simulation of colliding elastic objects

Rotations and internal vibrations are phenomena which absorb energy and therefore overshadow the underlying principles. In order to detect and to understand these basic principles the collision experiments should be designed in such a way that rotation is avoided and that the effects caused by internal vibrations can be neglected.

To avoid rotational effects head-on collisions are necessary



Such a head-on collision is shown in the simulation "7-head-on-collision". If the stiffness of all the springs are set to higher values it can be observed, how internal vibrations can be minimized.

When the stiffness is high enough, an object can be idealized as rigid and internal vibrations can be neglected.

Nevertheless the object has to be completely elastic during the collision process. The characterization „rigid body“ therefore has to be taken as „very very elastic but very very stiff“.

Fig 7.3.: Simulation "7-head-on-collision"

When the spring constant D is increased it can be observed that the internal vibration can finally be neglected.

Interesting settings for D (spring constant) are $D = 10; 50; 100; 500; 1000; 2000$ units.

About the user interface

The spring constant D for all existing springs can be set in the "Spring Properties" window, to be called from a submenu of the Window menu (main menu line of the JavaXYZ-cube). To apply the actual value to all existing springs, the related Button has to be activated. .

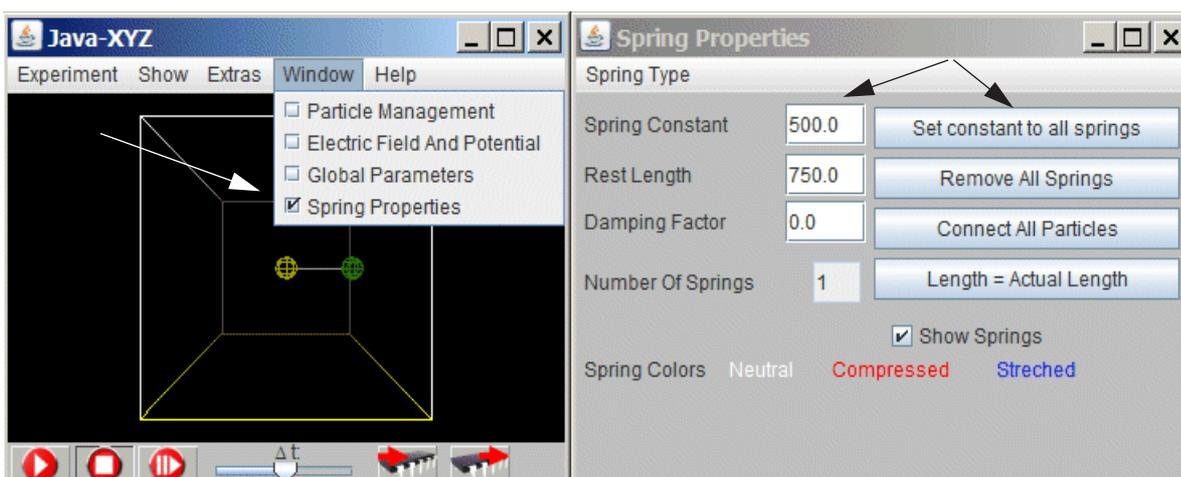


Fig 7.4.: "Spring Properties" window where a common spring constant can be applied to all existing springs

7.3. Collision with Idealistic Rigid bodies - Conservation of Momentum

The mathematical treatment of collision processes is based on the model of the rigid body as an idealization, where internal vibrations can be completely neglected.

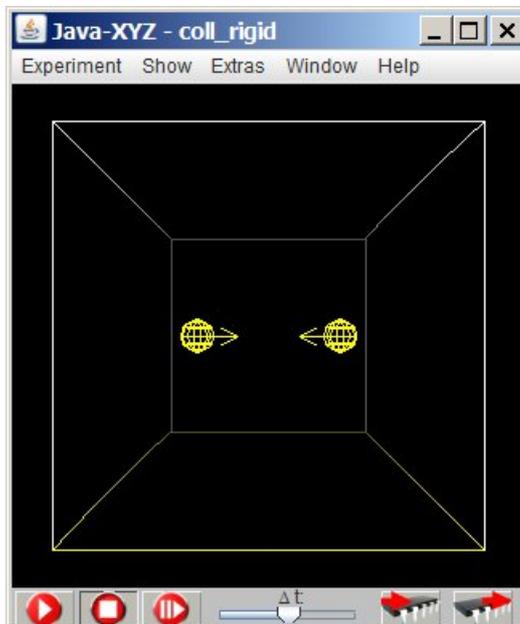
The particles in the following simulation are modelled as rigid objects. This means that the elastic deformation during collision does not use up energy and that no internal vibrations can exist.

The collisions with the walls of the cube, however, are modelled as completely inelastic. This means that during collision with the walls all energy is absorbed.

The simulation "7-collision-rigid-objects" allows to study collision processes by changing the x-component of the velocity of each particle as well as their mass.

To explain the outcome of such collision processes, two conservation laws of general validity have to be accepted: the conservation of energy and the conservation of momentum.

The derivation of these laws and the definition of energy and momentum is found in any standard textbook.



Normally, when two particles collide, their velocities are changing. This implies that for every single particle its momentum $p = m v$ is also changing. However, it will be shown that the total momentum, the sum of both momentums, is constant in every moment in time, in other words: momentum is conserved.

In the given simulation (as mentioned above) there is an exception when a particle collides with the walls of the cube. These walls are modelled as completely inelastic and the cube as having an infinitely large mass. Therefore the cube can absorb any momentum of the particle by moving with an infinitely small velocity to the opposite side.

Fig 7.5.: Simulation "7-collision-rigid-objects"

The following table shows the result from some collision processes to demonstrate that momentum is conserved.

To add momentum, it has to be known, that momentum is a vector.

When defined for one fixed directions, the 2 possible orientations are indicated as positive (+) or negative(-).

The total momentum of 2 particles with $+p_1$ and $-p_1$ is $p = +p_1 + -p_1 = 0$. The procedure is the same as if velocities with opposite orientations have to be added.

In the following table some collision processes with different settings have been documented. More such processes can and should be entered to prove the law about conservation of momentum.

t_1 stands for a moment in time before and t_2 after collision. The numbers are given in suitable units.

| # | m_1 | v_1 at t_1 | p_1 at t_1 | v_1 at t_2 | p_1 at t_2 | m_2 | v_2 at t_1 | p_2 at t_1 | v_2 at t_2 | p_2 at t_2 | p_1+p_2 at t_1 | p_1+p_2 at t_2 |
|-----|-------|----------------------|----------------------|----------------------|----------------------|-------|----------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|
| 1 | 1 | 100 | 100 | 0 | 0 | 1 | 0 | 0 | 100 | 100 | 100 | 100 |
| 2 | 1 | 100 | 100 | -100 | -100 | 1 | -100 | -100 | 100 | 100 | 0 | 0 |
| 3 | 3 | 100 | 300 | 50 | 150 | 1 | 0 | 0 | 150 | 150 | 300 | 300 |
| 4 | 3 | 100 | 300 | 0 | 0 | 1 | -100 | -100 | 200 | 200 | 200 | 200 |
| 5 | 2 | 100 | 200 | ? | ? | 1 | 0 | 0 | ? | ? | 200 | ? |
| 6 | 2 | 100 | 200 | ? | ? | 1 | -100 | -100 | ? | ? | 100 | ? |
| 7 | 1 | 100 | 100 | | | 1000 | -100 | -10^5 | ? | ? | ? | ? |
| ... | | | | | | | | | | | | |

Fig 7.6.: measurement protocol of collision processes

Basic law:

The total momentum of a system of interacting particles is conserved.

8. Acceleration and Deceleration through Gravity

8.1. How can a Satellite be Accelerated by a Planet?

Many times it is reported in newspapers and on TV that a satellite first passes a nearby planet to increase its speed before it is heading towards its final destination. How is this gain of momentum possible?

Why is a satellite accelerated more when it is approaching a planet than it is decelerated when it departs after the fly by?

The general law of gravity is indeed one of the most mysterious laws of nature, which is known to us. However, there is no doubt that the attractive force between two objects does not depend on the direction in space.

The *Simulation "8-planet-satellite"* offers the possibility to study the question posed above in detail.

In this simulation two spherical bodies (satellite and planet) are shown with a mass ratio of $1/200000$. Mutual gravitation is simulated.

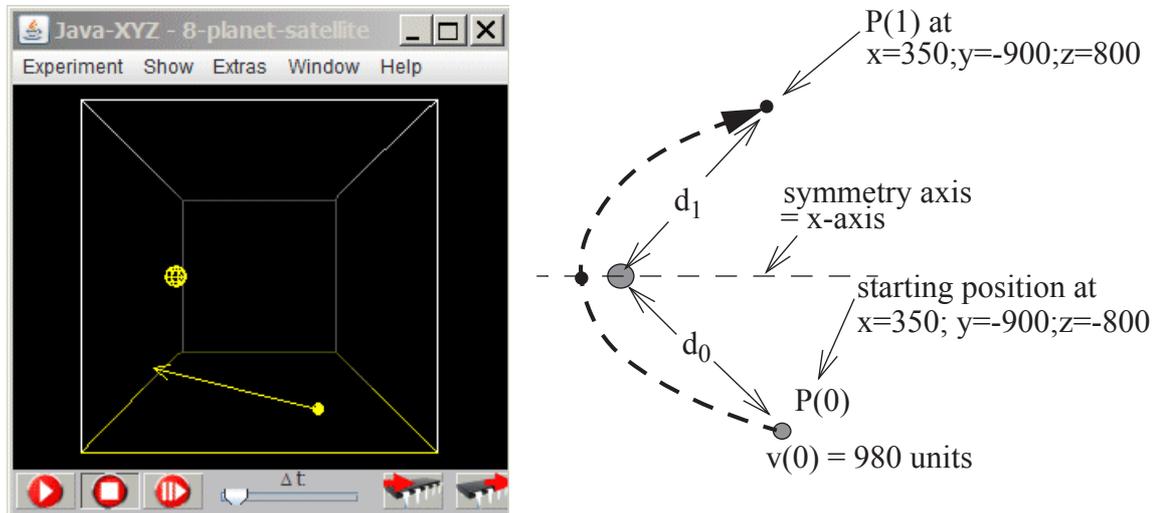


Fig 8.1.: Simulation "8-planet-satellite"

First case: planet at rest

In this case no gain in momentum can be expected because of the spherical symmetry of the gravitational forces. The gain in velocity, while approaching, is balanced by the lost in velocity when departing. In agreement with this statement the simulation shows, that the satellite - starting at $P(0)$ with $v(0) = 980$ units - has the same speed when it has reached the point $P(1)$ (fig 8.1).

Remark: In order to accomplish a precise measurement, the simulation can be advanced stepwise to the point $P(1)$, at which after the fly by the same distance from the planet has been reached as at the starting point $P(0)$.

The data for the satellite can be monitored during its fly by, by activating the monitor button in the "Particle Inspector" window"

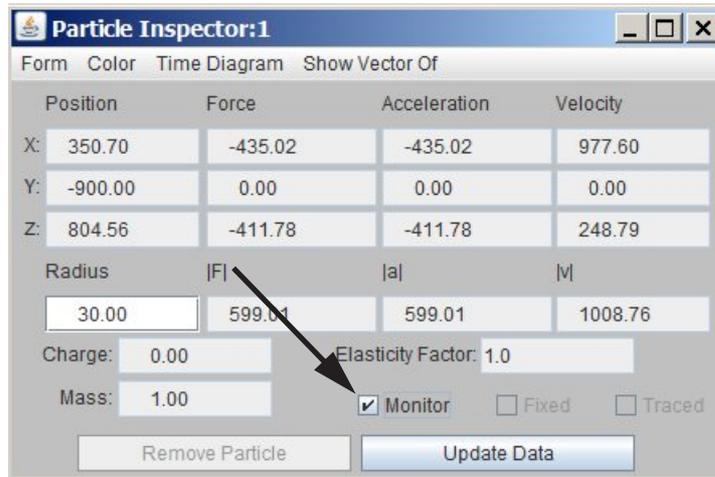


Fig 8.2.: How to activate to monitor data of a selected particle

Second case: planet is moving

The situation is changed when the planet is not at rest but moving. The direction of this movement defines a symmetry axis. Depending on its movement relative to this axis, the satellite will either speed up or slow down after having passed the planet.

Question: Assume that in the situation shown in the figure above the planet moves along the x-direction to the right. Will the planet, starting at $P(0)$ with a distance d_0 from the planet, speed up or slow down after having passed the planet and reached the same distance $d_1 = d_0$ from the planet?

When looking for an answer it may be helpful to consider that the total energy and the total momentum of the system planet/satellite must be conserved.

8.2. Conservation of Energy and Momentum

When will the satellite accelerate, when decelerate?

This question can be answered on the basis of the laws of conservation of energy and momentum.

From the law about conservation of energy it follows that the planet must slow down so that the satellite can move faster or vice versa.

To draw a conclusion from the law of conservation of momentum two cases have to be considered.

- 1. When approaching the planet the satellite moves in the opposite direction as the planet. In this case the total momentum p is smaller than the momentum of the planet.
- 2. When approaching the planet the satellite moves in the same direction as the planet. In this case the total momentum p is larger than the momentum of the planet.

For both cases it is assumed that the momentum of the planet is defined as positive.

In case 1 the total momentum of the system is being increased due to the change of direction of the satellite after having passed the planet. Conservation of momentum can only be guaranteed if the velocity of the planet is reduced accordingly. And conservation of energy implies that the velocity of the satellite must increase.

In case 2 the total momentum of the system would decrease due to the change of direction of the satellite after having passed the planet. Conservation of momentum can only be guar-

anteed if the velocity of the planet is increased accordingly. And this implies that the velocity of the satellite must decrease.

This rather abstract explanation can be enriched by a more concrete one by making use of a simulation.

8.3. Acceleration of a Satellite caused by a Planet

The simulation "8-planet-satellite.acceleration" shows again two spherical objects (satellite/planet) with a mass ratio of 1/200000 and mutual gravitational attraction. The planet moves with a velocity of 100 units in the positive x-direction

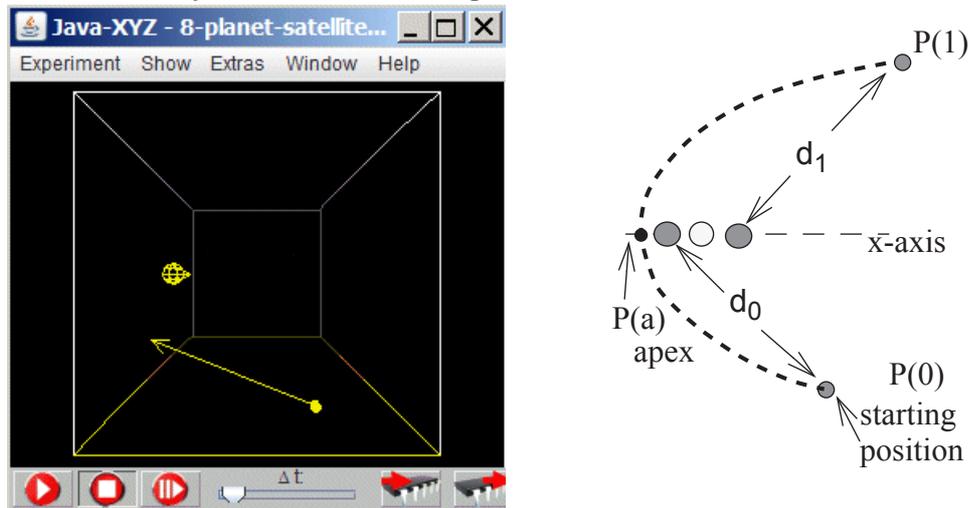


Fig 8.3.: Simulation "8-planet-satellite-acceleration"

The simulation shows that the velocity of the satellite is nearly increased by 200 units, if it has reached after flyby the point P(1) with the same distance to the satellite as at P(0). If, however, the planet moves with $v = -100$ units (in the negative x-direction), the velocity of the planet is decreased by nearly 200 units when having reached again the same distance to the planet as at the starting position (simulation "7-satellite-deceleration").

Remark: In order to accomplish a precise measurement, the simulation can be advanced stepwise when approaching the apex P(a) or point P(1).

Explanation

The path of the satellite when passing the planet is hyperbolic.

The simulation is set-up in such a way that this hyperbolic path is symmetrically oriented in respect to the x-axis.

When the satellite has reached the apex P(a) the distance between satellite and planet has reached a minimum. Until it has reached this point, it is approaching the planet. When it has passed this point the distance is increasing.

It follows that at the point P(a) the velocity component of the satellite in the direction of the planet - in this case the x-direction - must be the same as the velocity of the planet in the x-direction, thus nearly 100 units.

The x-component of the starting velocity of the satellite $v_x(0)$ is therefore increased by the amount $(v_x(0) + 100)$ units and this during flying time from P(0) to P(a).

For symmetry reasons we can conclude that the same velocity change will occur during the flight from P(a) to P(1) where the satellite has reached the same distance to the planet as before at P(0).

The simulation shows that the x-component at $P(1)$ is about 200 units larger than at $P(0)$.
By approaching stepwise the point $P(a)$ you can check that the x-component of the satellite velocity takes the value +100 or -100 depending on the orientation of the planet velocity.
And certainly the simulation shows that the speed of the satellite is reduced, compared with the starting velocity, if the planet moves in the negativ x-direction.

9. Exercises with Satellite Orbits

9.1. Introduction

Spatial imagery is an important capability and a prerequisite to solve many different problems in physics, either in so-called „Gedanken-experiments“ or those where moving objects are involved.

The following videos and assignments are offered as some kind of exercise material, assuming that spatial imagery as any other capability can be further developed by repetitive training.

9.2. Satellite Orbits in 3d and 2d

The video (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/sat_polar.htm) shows a satellite moving around the earth while its projection perpendicular to the surface of the earth is drawn.

Once a satellite is in orbit, the orientation of the plane in which it is orbiting, is stable in space while the earth is rotating once in 24 hours around its polar axis. Therefore the projection of the satellite on the surface of the earth will look like some kind of displaced circles. The size of the displacement will depend on the ratio of the time period for one rotation of satellite and earth.

In 3d this process looks kind of simple. The task is now to imagine, which curve will show up, if the spherical surface of the earth is represented in 2d as a rectangular map.

The video shows this transformation from 3d to 2d as a continuous process and may help to see and understand the relation between these two different representations of the identical process.

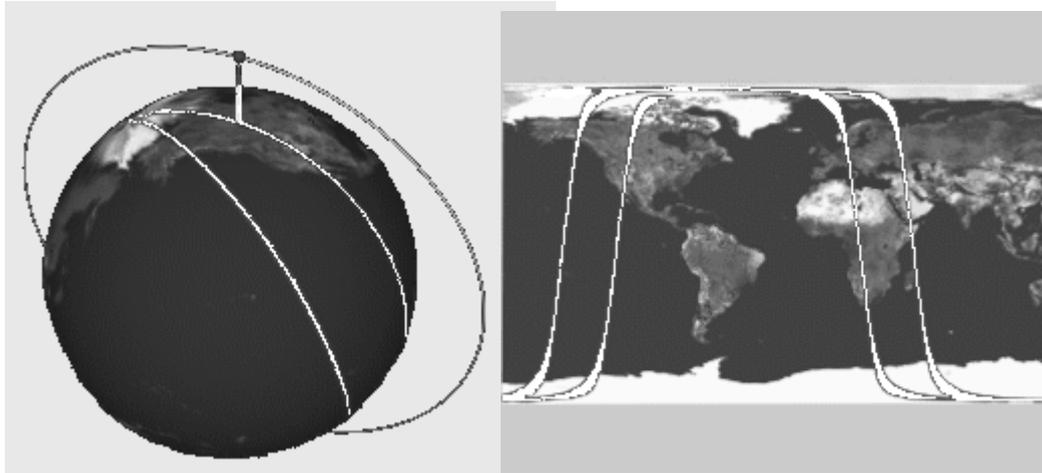


Fig 9.1.: Simulation about a polar satellite

9.3. Orbit of the ISS

In the last video the satellite was crossing over the north and south pole. The polar axis of the earth and the orbit of the satellite were therefore lying within the same plane.

The ISS - International-Space-Station - however, does not follow this orbit. There is a distinct angle between the plane of its orbit and the polar axis of the earth.

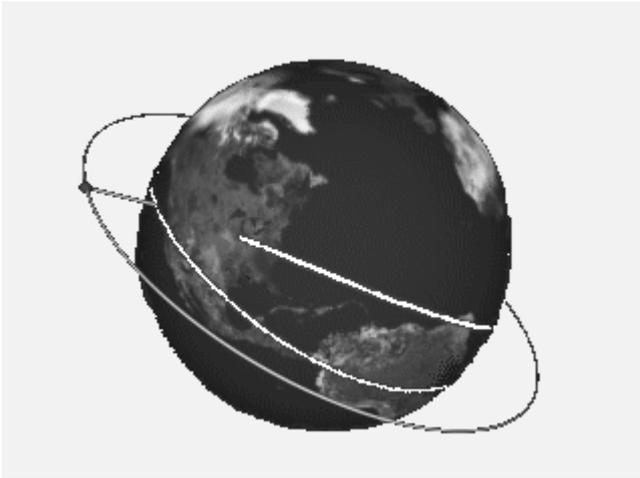


Fig 9.2.: Video about a non-polar satellite

Try to imagine first, which kind of trace will occur when the position of the satellite is projected onto the 2-dimensional worldmap and play the video (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/non_polar_sat.htm) to check your assumptions.

9.4. Geostationary Satellite

In general the orbit of a satellite has the form of an ellipse with the earth at one of its focus points. A special situation has been reached for the following conditions:

- The orbit of the satellite lies in the same plane as the equator of the earth.
- The orbit is not an ellipse but a circle.
- The radius of the orbit is so large that it takes 24 hours to complete one round.

The satellite under these conditions is geostationary, which means that its projection on the surface of the earth does not move but remains on the same spot.

For a computation of the radius for a geostationary satellite see the paragraph "Computation" below.

When looked at from the earth a geostationary satellite appears under a constant angle and can therefore serve for instance as transmitter of TV programs.

Look at the video (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/geo_1_eng.htm) and try to find the answer to the posed question, before you play the next video (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/geo_2_eng.htm) to check your assumption .

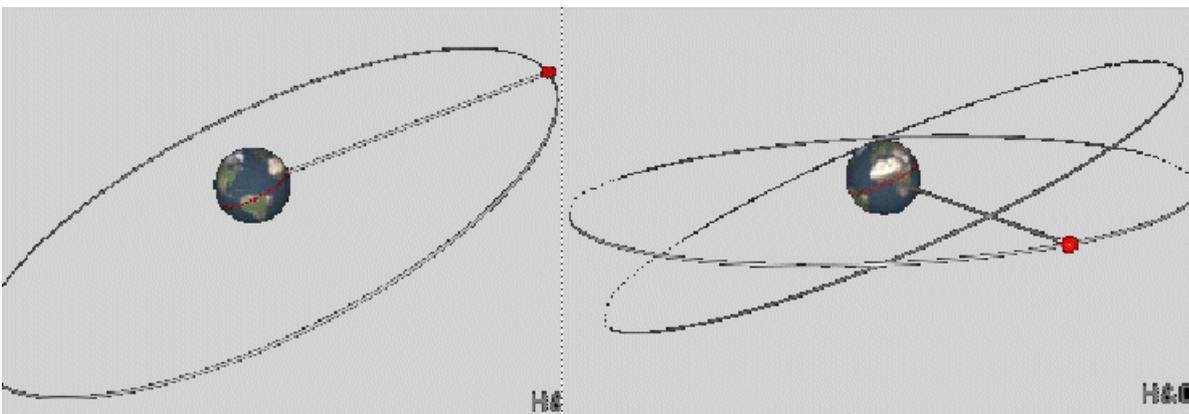


Fig 9.3.: Video about a geostationary satellite

It may be a good exercise for training spatial imagery to build a model with a globe and a satellite and to see, how this curve as projection on the surface of the globe does develop. You may also try to find out which curve will develop, if the orbit of the geostationary satellite is deformed slightly to an ellipse.

Computation of the Radius for a Geostationary Satellite

A geostationary satellite has to complete a full circle in 24 hours.

The gravitational interaction between earth and satellite has to be equal to the centripetal force needed for a circular movement. With this condition the radius R of the orbit of a geostationary satellite can be calculate as follows.

$$F_{\text{gravitation}} = \gamma \cdot \frac{m_1 \cdot m_2}{R^2}$$

$$F_{\text{centripetal}} = m_1 \cdot \frac{v^2}{R}$$

$$\gamma \cdot \frac{m_1 \cdot m_2}{R^2} = m_1 \cdot \frac{v^2}{R} \quad v = \frac{2\pi \cdot R}{t}$$

$$\gamma \cdot \frac{m_2}{R^2} = \frac{4\pi^2 R}{t^2} \quad R^3 = \frac{\gamma \cdot m_2 \cdot t^2}{4\pi^2}$$

$$\gamma = 6,7 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg s}^2} \quad m_2 = 6 \cdot 10^{24} \text{kg} \quad t = 60 \cdot 60 \cdot 24 = 86400 \text{s}$$

$$R^3 = \frac{6,7 \cdot 6 \cdot 75}{4 \cdot 9,8} 10^{-11+24+8} = 76,9 \cdot 10^{21} \text{m}^3$$

$$R \sim 4,3 \cdot 10^7 \text{m} = 43000 \text{km};$$

$$R_{\text{earth}} \sim 6300 \text{ km}; \text{ Distance}_{\text{Earth surface - satellite}} \sim 36700 \text{ km}$$

9.5. Molniya Satellite

On the upper northern hemisphere the normal geostationary satellites are not visible. The video (http://www.astrophysik.uni-kiel.de/~hhaertel/CGA_e/Flv/molnya_eng.htm) shows how the orbits of the so-called Molniya satellites have been selected to serve during parts of their orbit as nearly geostationary satellites.

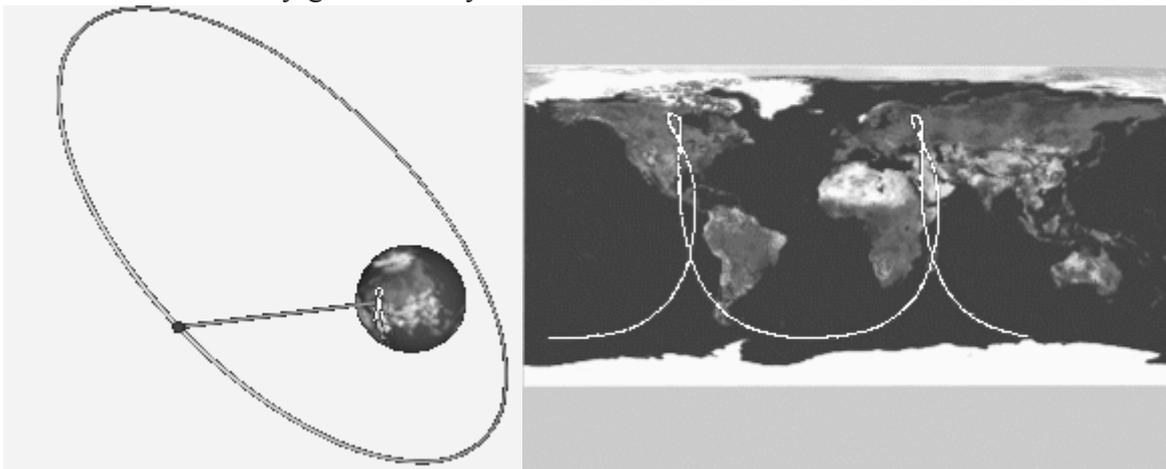


Fig 9.4.: Video about a Molniya satellite

Three of such satellites in a row are sufficient to serve for a continuous transmission.

10. Oscillating Motion

10.1. General Properties

Every system which can perform an oscillating motion, has two general properties:

- 1. It has an equilibrium position.
- 2. When the system leaves this position, restoring forces seek to drive the system back to equilibrium.

10.2. Harmonic Oscillator

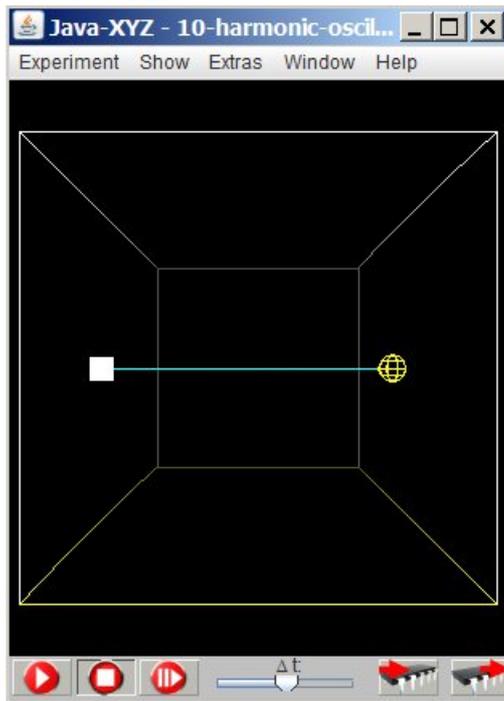
A special case of an oscillating movement is defined as harmonic if the restoring forces are proportional to the distance from the equilibrium position.

According to Hook's law the following proportion exists between the elongation of a spring s and the resulting force:

$$F = D s \quad (D = \text{spring constant})$$

This linear relation is in fact the condition stated above for the existence of an harmonic oscillation. A spring is therefore capable of driving an harmonic oscillation.

The simulation „10-harmonic-oscillation“ demonstrates such an oscillation, driven by a spring. An important parameter for an oscillation is the time period for one cycle, indicated as T .



When changing D and m , preferably in a systematic manner, the dependence of T on m and D (spring constant) can be derived and should, when ever possible, be checked by corresponding real experiments.

For further explanations see next chapter.

The mass of the oscillating object can be change in the „Particle Inspector“ window.

The spring constant D can be changed for a selected spring in the "Spring Editor" window.

Fig 10.1.: Simulation "10-harmonic-oscillation"

$$D \cdot A = \frac{m \cdot v^2}{A} \quad A = \sqrt{m/D} \cdot v$$

$$v = \frac{A}{\sqrt{m/D}} \quad T = \frac{2\pi A}{v} = 2\pi\sqrt{m/D}$$

This proves in accordance with the simulation "10-harmonic-oscillation" the relation: $T \sim \sqrt{m/D}$.

This relation also proves that the time T needed for one cycle is independent of the amplitude and therefore also independent of the velocity with which an object starts from the equilibrium position.

10.4. Harmonic Oscillator - Amplitude and Period

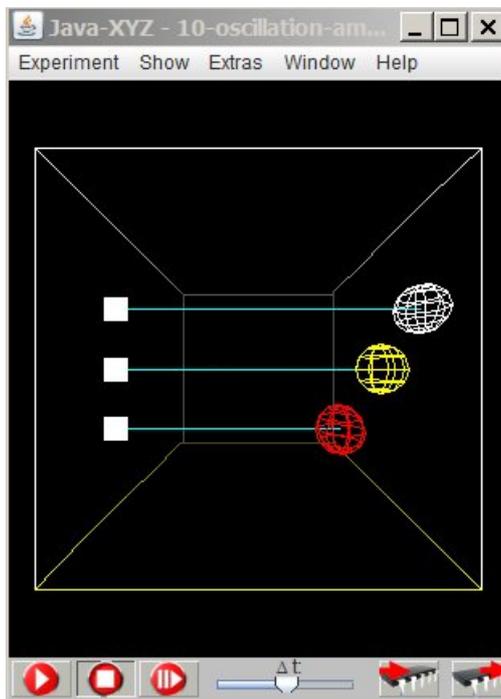


Fig 10.4.: Simulation "10-oscillation-amplitude"

As stated above, the time T needed for an oscillation is independent of the velocity with which an object starts from the equilibrium position.

Although this is exactly true only for harmonic oscillators, we can use this principle to approximate the motion of other vibrating objects.

If a wine glass is struck, for example, it starts oscillating and transfers this motion to the surrounding air. We hear these vibrations as a sound with a higher or lower pitch, depending on how fast the glass is vibrating.

The pitch, which is related to the duration of one oscillation, is the same, regardless of the force, with which the glass was struck. A specific glass (with the same content) will always produce a tone of the same pitch.

In general: The time period for one oscillation T or its frequency f (number of oscillations per time unit) does not depend on the amplitude.

This time period T is a property of the system, determined by the mass m of the oscillating object and the constant D of the restoring forces.

The simulation "10-oscillation-amplitude" demonstrates this statement.

10.5. Movement of a Pendulum

The movement of a pendulum is interesting under several different aspects.

- For a long time this movement was used to divide the continuous flow of time into equal intervals and thus to run clocks.
- The restoring forces, driving the pendulum back and forth, come close to the condition, which is set up for harmonic oscillations.

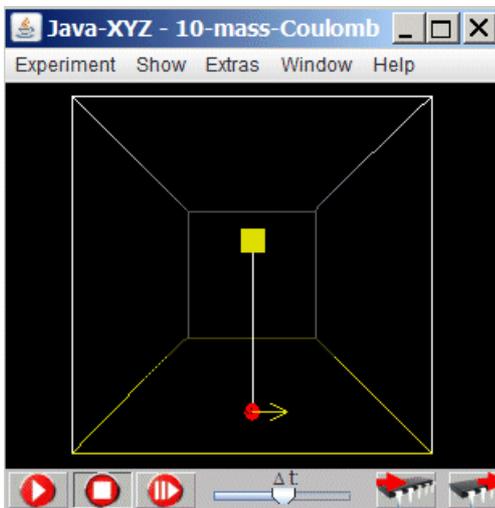
- Since a pendulum moves along a circle, centripetal forces are involved.
- A pendulum is driven by gravity, and therefore certain analogies to the motion of a freely falling body can be found.

All these aspects make the pendulum to an interesting object for studying these laws and phenomena in an integrated manner.

10.6. Mass of the Bob m and the Period for one Oscillation T

The two simulations "10-mass-Coulomb" and "10-mass-gravitation" are both simulating a movement of a pendulum. They differ in the kind of the acting force. In the first simulation the force is produced by an electrostatic field, oriented downwards (in $-z$ -direction) acting on a charged bob. In the second simulation a gravitational field is acting on a neutral bob. Interesting and instructive is the difference which shows up, if the mass of the bob is changed. In the first simulation the mass has a strong influence on the period of the pendulum, in the second simulation the period is independent of the mass.

Remark:



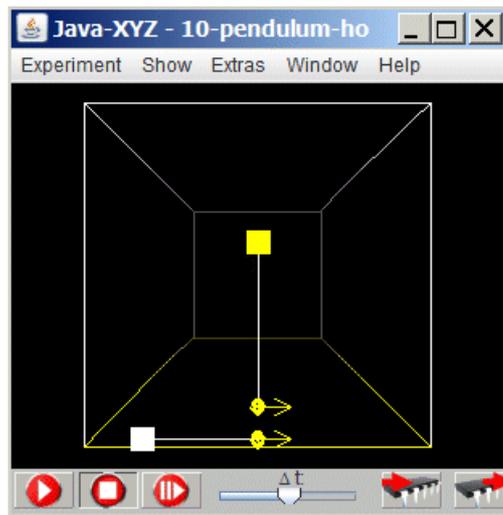
The tiny trembling motion, which appears with larger changes of the mass are due to the fact, that with such changes the elastic suspension is no longer in equilibrium between the gravitational force downwards and the centripetal force upwards.

Fig 10.5.: Simulation "10-mass-Coulomb" or "10-mass-gravitation"

The explication, why the period of a pendulum is independent of the mass of the bob is given by the influence of the gravitational mass. In case of a gravitational pendulum the gravitational mass compensates the influence of the inertial mass. In a pure electrostatic pendulum, however, the gravitational mass is missing and the always existing inertial mass shows its influence.

The fact that the period of a pendulum is independent of the mass of its bob can easily be checked with rather simple experimental means.

10.7. Pendulum as „Harmonic Oscillator“



The simulation "10-pendulum harmonic-oscillator" allows to compare the movement of a pendulum and an harmonic oscillator.

In the starting position the setup is done in such a way that both movements are very similar.

A pendulum therefore behaves nearly like an harmonic oscillator.

Fig 10.6.: Simulation "10-pendulum-ho"

However, there is an interesting difference in respect to the influence of the mass of the swinging bodies.

If the mass of the swinging objects is changed, an important difference in behaviour shows up. Small changes of 20% or 40% are sufficient to see the difference.

Result: While the period T of the harmonic oscillator depends on the mass of the swinging objects, the period for the pendulum does not. Why?

The explanation can be found in the different type of force which drives the oscillating movement.

For the harmonic oscillator the elastic force of the spring remains the same when the inertial mass is changed.

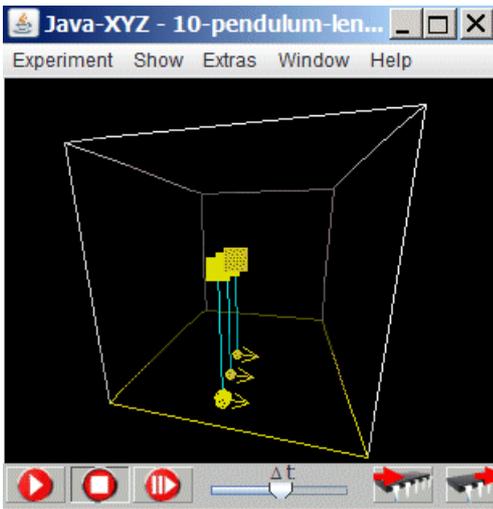
For a pendulum the driving force is the gravitational attraction of the earth. This force is increased proportional to the inertial mass of the bob, because inertial and gravitational mass are strictly proportional. An enlarged inertial mass and an enlarged force neutralize themselves mutually and this is the reason why the period of the pendulum is independent of the mass of the bob.

10.8. Pendulum Period T and Length of Suspension

It was an important fact for building pendulum clocks, that the accuracy of the clock did not depend on the mass of the bob. Therefore the form and mass of this bob could have any size and was free to be designed at will.

How could the accuracy of such a clock be changed, if it proceeded or followed?

The answer: By changing the length of the suspension.



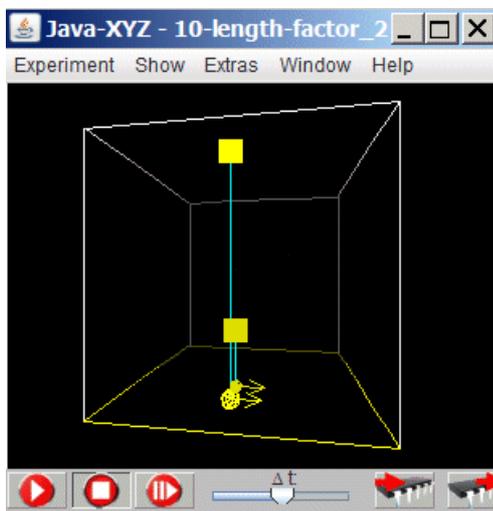
The prepared simulation "10-pendulum-length" shows three pendula which differ in length of the suspensions by 5% and 10% respectively.

Clearly the periods are different. This effect can quite easily be checked by a real experiment.

The relation between the time for a full swing, the period T , and the length of the suspension l can be derived from theoretical considerations as: $T \sim \sqrt{l}$

The simulation "10-pendulum-different-length" demonstrates this kind of relation.

Fig 10.7.: Simulation "10-pendulum-length"



The ratio of the length of the 2 suspensions is $l_1 / l_2 = 4$.

The ratio of the periods is $T_1 / T_2 = \sqrt{4} = 2$.

Fig 10.8.:
Simulation "10-pendulum-different_length"